

# Theory of dipolar polaritons in microcavity-embedded coupled quantum wells in electric and magnetic fields

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**Abstract.** We present a precise calculation of spatially-indirect exciton states in semiconductor coupled quantum wells and polaritons formed from the coupling of an indirect exciton to the optical mode of a microcavity. We include the presence of electric and magnetic fields applied perpendicular to the QW plane. Our model predicts the existence of polaritons with a large dipole moment, known as dipolaritons. We discuss the electric and magnetic field dependence of such states.

## Introduction

Exciton-polaritons in microcavities in the strong coupling regime have been studied intensively in recent years. They can be realized by placing semiconductor quantum wells (QWs) at the antinode position of a resonant electromagnetic field inside a cavity. Notable advances include the demonstration of polariton condensation [1] and room temperature lasing [2]. Spatially indirect excitons, formed from an electron and a hole separated into adjacent coupled quantum wells (CQWs) by an external bias are also an attractive system for study. This is due to the possibility to control the exciton lifetime and macroscopic static dipole moment by varying the externally applied electric field.

Recently, CQWs were embedded in a planar Bragg-mirror microcavity to create polaritons with a large dipole moment typical for indirect excitons [3]. Usually, an indirect exciton is only weakly coupled to light due to the reduced overlap of the electron and hole wave functions. However, when asymmetric CQWs are used, the electric field can be chosen such that the indirect exciton, direct exciton and the cavity mode are brought into resonance, resulting in a strongly coupled three-level system. Such hybrid quasiparticles, called dipolaritons, present a system with a greater flexibility of control and new possible applications compared to polaritons formed from direct excitons. Applying a magnetic field offers a further degree of control. Here, we employ a rigorous calculation of polariton states in microcavity-embedded CQWs to examine the electric and magnetic field dependence of the properties of dipolaritons.

## 1. CQW excitons in electric and magnetic fields

We first seek eigen states of the Hamiltonian for an exciton under external bias and magnetic field, given by

$$\hat{H}(\mathbf{r}_e, \mathbf{r}_h) = \hat{H}_e(\mathbf{r}_e) + \hat{H}_h(\mathbf{r}_h) - \frac{e^2}{\epsilon_b |\mathbf{r}_e - \mathbf{r}_h|} + E_g, \quad (1)$$

with

$$\hat{H}_{e,h}(\mathbf{r}) = \hat{p}_{e,h}(\mathbf{r}) \frac{1}{2} \hat{m}_{e,h}^{-1}(z) \hat{p}_{e,h}(\mathbf{r}) + U_{e,h}(z), \quad (2)$$

$$\hat{p}_{e,h}(\mathbf{r}) = \frac{\hbar}{i} \nabla_{\mathbf{r}} \pm \frac{e}{c} \mathbf{A}(\mathbf{r}), \quad (3)$$

where  $\mathbf{r}_{e,h}$  are the electron and hole coordinates. For a multi-layered heterostructure, we adopt a cylindrical coordinate system  $(\rho, \phi, z)$  with the  $z$ -axis along the QW growth direction. The electron and hole effective mass tensors,  $\hat{m}_{e,h}(z)$ , are composed of in-plane and perpendicular components,  $m_{e,h}^{\parallel}$  and

$m_{e,h}^{\perp}(z)$ , respectively. We neglect any  $z$ -dependence of  $m_{e,h}^{\parallel}$ , which is justified by low mass contrast in the heterostructures treated here and a minor contribution of the electron and hole wave functions outside the well regions.  $m_{e,h}^{\perp}(z)$  are layer-dependent step functions, as are the QW confinement potentials  $V_{e,h}^{\text{QW}}(z)$ . The confinement and the external bias are included in the potentials  $U_{e,h}(z) = V_{e,h}^{\text{QW}}(z) \pm eFz$  where  $F$  is the electric field. For the band gap  $E_g$  and in-plane masses  $m_{e,h}^{\parallel}$ , values for the QW layers are used.  $\epsilon_b$  is the average permittivity of the structure. For a magnetic field  $\mathbf{B}$  along the growth direction, one can take the vector potential, using the symmetric gauge, in the form  $\mathbf{A}(\mathbf{r}) = \frac{1}{2} \mathbf{B} \times \mathbf{r}$ .

Optically active eigen states of the Hamiltonian (1) have zero angular momentum and their wave functions,  $\Psi(\mathbf{r}_e, \mathbf{r}_h)$  are found by separating the in-plane center of mass and relative coordinates ( $\mathbf{R}$  and  $\boldsymbol{\rho} = (\rho, \phi)$ , respectively) using the substitution [4],

$$\Psi_k(\mathbf{r}_e, \mathbf{r}_h) = \psi_{\mathbf{P}}(\boldsymbol{\rho}, \mathbf{R}) \varphi_k(z_e, z_h, \rho), \quad (4)$$

$$\psi_{\mathbf{P}}(\boldsymbol{\rho}, \mathbf{R}) = \exp\left(i \left[ \mathbf{P} + \frac{e}{c} \mathbf{A}(\boldsymbol{\rho}) \right] \cdot \frac{\mathbf{R}}{\hbar} \right). \quad (5)$$

Here, the index  $k$  labels the exciton quantized states,  $\mathbf{P}$  is the in-plane center-of-mass momentum and  $z_{e,h}$  are the electron (hole) coordinates in the growth direction.  $\varphi_k(z_e, z_h, \rho)$  describes the internal structure of the exciton and is computed using a multi-sub-level approach [5, 6, 7]. This entails expanding the wave function into the basis of Coulomb-uncorrelated electron-hole pair states,

$$\varphi_k(z_e, z_h, \rho) = \sum_n \Phi_n(z_e, z_h) \phi_n^{(k)}(\rho). \quad (6)$$

Each pair state,  $\Phi_n(z_e, z_h)$ , is the product of single particle electron and hole wave functions which themselves are eigen states of the perpendicular motion Hamiltonians,

$$\hat{H}_{e,h}^{\perp}(z) = -\frac{\hbar^2}{2} \frac{\partial}{\partial z} \frac{1}{m_{e,h}^{\perp}(z)} \frac{\partial}{\partial z} + U_{e,h}(z). \quad (7)$$

We calculate the radial components of the wave function  $\phi_n^{(k)}(\rho)$ , using a matrix generalization of the shooting method with Numerov's algorithm incorporated in the finite difference scheme. The full solution (6) enables us to determine the exciton absorption spectrum as a function of  $F$  and  $B$  and to calculate the exciton dipole moment, Bohr radius, oscillator strength, binding energy and magnetic field induced effective

mass enhancement. Full details can be found in Ref. [6] where we analyze the dependence of these quantities on the indirect-direct exciton crossover which occurs for changing  $B$  and/or  $F$ .

## 2. CQW polaritons in external fields

Using the scattering matrix method [8], we solve coupled Maxwell's and material equations in a microcavity with embedded CQWs for the light field  $\mathcal{E}(z)$ , expressed by the following integro-differential wave equation,

$$\left(K^2 - \frac{\partial^2}{\partial z^2}\right) \mathcal{E}(z) = \frac{\omega^2}{c^2} \left[ \epsilon_b \mathcal{E}(z) + 4\pi \int \chi(z, z') \mathcal{E}(z') dz' \right], \quad (8)$$

where  $K = |\mathbf{P}|/\hbar$  and  $\omega$  is the frequency of light. Our accurate description of the exciton states is incorporated using a non-local susceptibility  $\chi(z, z')$  which describes the exciton polarizability in the active (QW) layers,

$$\chi(z, z') = e^2 d_{cv}^2 \sum_k \frac{\varphi_k(z, z, 0) \varphi_k(z', z', 0)}{E_k + \frac{\hbar^2 K^2}{2M_k^*} - \hbar\omega - i\gamma}. \quad (9)$$

Here,  $d_{cv}$  is the dipole matrix element,  $\gamma$  is a damping constant and  $E_k$  and  $M_k^*$  are the exciton energy and  $B$ -dependent effective mass, respectively. By solving the coupled equations (8–9), we determine the light field and polarization. For  $K = 0$ , the latter is expressed as,

$$\mathcal{P}(z_e, z_h, \rho) = \sum_k \frac{d_{cv}^2 \varphi_k(z_e, z_h, \rho)}{E_k - \hbar\omega - i\gamma} \int \mathcal{E}(z) \varphi_k(z, z, 0) dz. \quad (10)$$

The polariton brightness  $\mathcal{F}$  and dipole moment  $\mathcal{D}$  are,

$$\mathcal{F} = \frac{1}{\mathcal{N}} \left| \int \mathcal{P}(z, z, 0) dz \right|^2, \quad (11)$$

$$\mathcal{D} = \frac{2\pi}{\mathcal{N}} \int \int \int |\mathcal{P}(z_e, z_h, \rho)|^2 (z_e - z_h) \rho d\rho dz_e dz_h, \quad (12)$$

$$\mathcal{N} = 2\pi \int \int \int |\mathcal{P}(z_e, z_h, \rho)|^2 \rho d\rho dz_e dz_h, \quad (13)$$

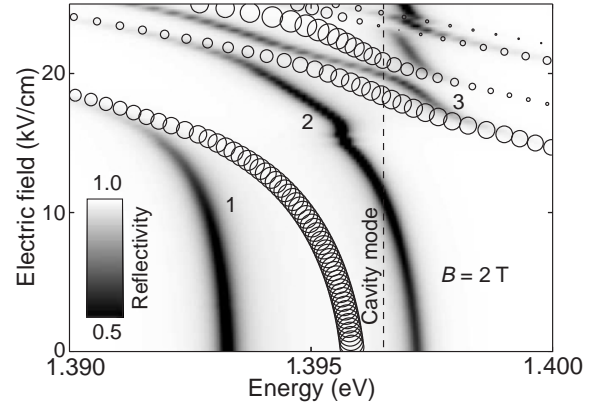
where  $\mathcal{N}$  is a normalizing constant.

We consider an InGaAs CQW structure inside a microcavity that was studied in Ref. [3]. Figure 1 shows the electric field dependence of the polariton reflectivity spectrum for  $B = 2$  T. The exciton states are shown by circles, each with an area proportional to the oscillator strength. The bare cavity mode is shown by a dashed line. Dips in reflectivity are the polariton states. The energies of the lowest three states are shown in Fig. 2a. The corresponding brightness and dipole moment are shown in Figs. 2b and 2c, respectively. Figures 2d–f show the same quantities as a function of  $B$  for  $F = 20$  kV/cm.

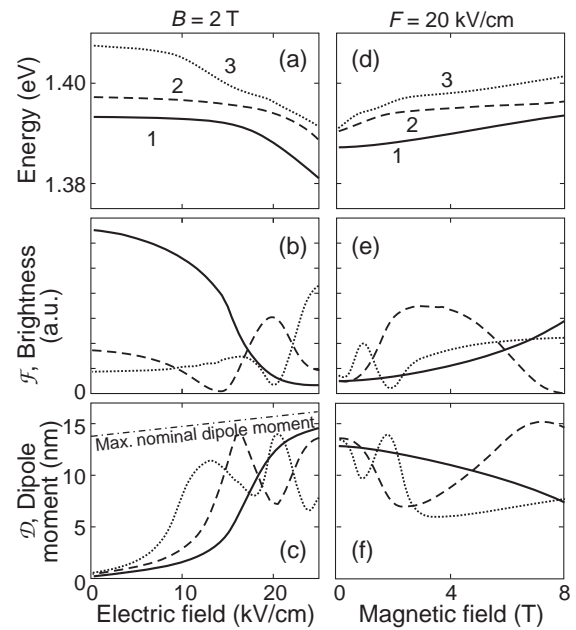
The model predicts the existence of dipolaritons characterized by a static dipole length that can be comparable to the nominal center-to-center distance of the CQWs and can be controlled by external fields (see Figs. 2c and 2f). An increase in the dipole length is accompanied by a decrease in brightness as the exciton transforms from a bright direct to a dark indirect state.

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**Fig. 1.** Electric field dependence of the polariton reflectivity spectrum in magnetic field (greyscale). Exciton states are shown by circles. The circle area is proportional to the oscillator strength. Data is calculated for 10-4-10 nm InGaAs microcavity-embedded asymmetric CQWs used in Ref. [3].



**Fig. 2.** Energy (a,d), brightness (b,e) and static dipole moment (c,f) of the lowest three polariton states as a function of  $F$  with  $B = 2$  T (a–c) and as a function of  $B$  with  $F = 20$  kV/cm (d–f). The states in (a–c) correspond to those labeled in Fig. 1.

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