OPTIMISING MATERIAL DENSITY OF CELLULAR BODIES IN HIGH ELASTIC DEFORMATIONS

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Summary For cellular bodies with uniform cell size, wall thickness, and shape, an important question is whether the same volume of material has the same effect when arranged as many small cells or as fewer large cells. Numerically, we assess the independent impact of cell size and number of cells on the mechanical behaviour of finite element models of periodic, honeycomb-like structures of neo-Hookean or Mooney-Rivlin material, and obtain that the mean non-linear elastic modulus and Poisson 's ratio in the cell walls increase as the number of cells increases while the total volume of solid material remains fixed. This is due to the enhanced elasticity of the cell walls when the material is distributed more uniformly throughout the structure. The results presented here provide valuable information in understanding the behaviour of cellular structures of non-linear elastic material and can be used to optimise their design for various applications.

INTRODUCTION

Cellular bodies are strong, pliable structures, built from seemingly fragile materials. Among the best known mechanical qualities of these structures are their high strength-to-weight ratio and energy absorption capacity, which arise from the inextricable relation between the geometric architecture and the non-linear elastic responses of their constituents. In many natural load-bearing structures, support requirements are typically met through a combination of increase in cell number or size and sustained sclerification (thickening and lignification) of the cell wall. For example, dicotyledon stems (*e.g.* magnolia, oak, sycamore) increase their diameter primarily by cell division which ultimately form the characteristic annual rings, while monocotyledon stems (*e.g.* lily, palm, reed) prevent mechanical failure through a combination of initiation of growth with a stem that is sufficiently wide for future supply and support demands, and increase in stem diameter and strength by sustained cell wall expansion and lignification, predominantly toward the stem periphery and base. Even though some monocot plants attain tree stature comparable with arborescent dicotyledons and conifers, their stems are relatively slender. By contrast, tall dicot trees have bigger stem diameters relative to their height than small trees, although the wood density representing the relative quantity of the cell wall in a given volume of wood tissue does not vary significantly among wood species. For living cellular structures, there are many physiological and ecological factors that affect their mechanical properties, and they also change over time. Nevertheless, for structures with uniform cell size, wall thickness, and shape, the fundamental question arises whether the same volume of cell wall material has the same effect when arranged as many small cells or as fewer large cells. In the case of small strain elastic deformations, thresholds on stiffness or strength can be set as constraints in the mechanical design or developmental process [1]. When large strains occur during functional performance, finding suitable criteria that account for the non-linear properties of the deforming cell walls is needed [2].

NON-LINEAR ELASTIC EFFECTS

In our study, finite element modelling was used to investigate the mechanical responses of periodic, honeycomb-like structures with regular geometry, such as square, diamond-shape, and hexagonal cells of non-linear hyperelastc material subject to large strain deformations. The models structures were created in SolidWorks, meshed in Gmsh, and imported into the FEBio software suite [3]. Each structure was made from a single piece of elastic material which occupied a thin square domain of (dimensionless) side 1 in the X- and Y- directions, and 0.1 in the Z-direction. Cells were equal size throughout the structure. The lower external horizontal face was free to slide in the X- and Z-directions, and fixed in the Y-directions. The upper external horizontal face was subject to a prescribed vertical stretch of 50% in the Y-direction, and was free to slide in the X- and Z-directions. The remaining external and internal cell faces were allowed to deform freely (see Figure 1).

For structures made from neo-Hookean or Mooney-Rivlin material, the numerical results show that the non-linear elastic modulus computed as the ratio between the mean effective Cauchy stress and the mean effective logarithmic strain and the non-linear Poisson's ratio defined as the negative ratio between the mean logarithmic strain in the horizontal and vertical direction, respectively, increase as the wall thickness increases while the number of cells is fixed, or as the number of cells increases while the total volume of solid material remains unchanged (see Figure 2). The mean value was calculated as the sum of the values on all the finite elements divided by the number of elements. These effects are due to the enhanced elasticity of the cell walls when more material is added or when the same material is distributed more uniformly throughout the structure.

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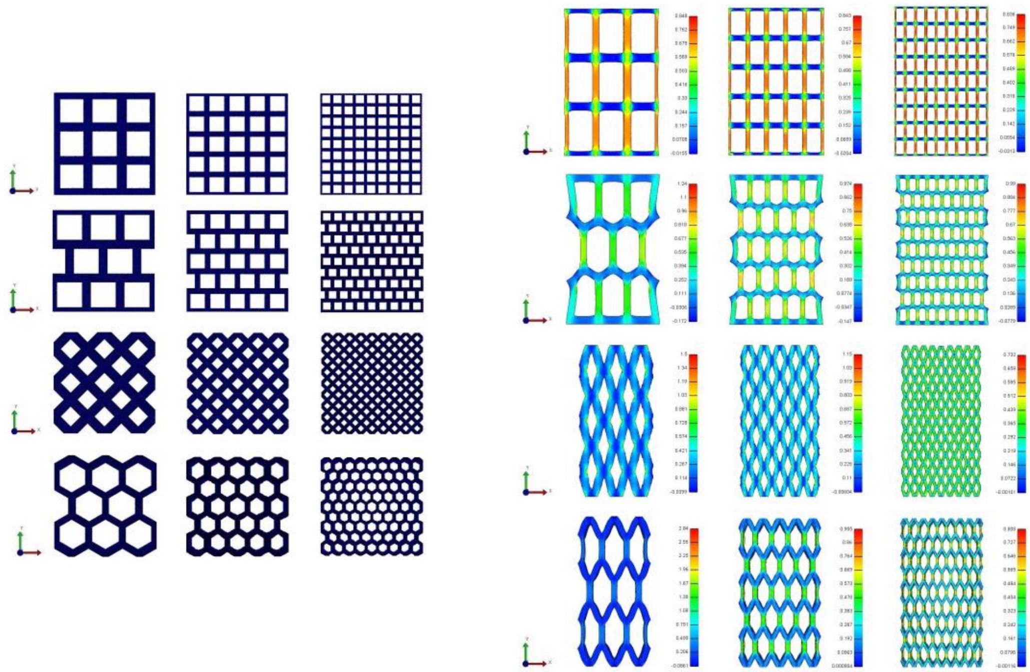

Figure 1. Undeformed model structures with different cell geometries and increasing number of cells while the volume of solid material is fixed (left) and deformed structures subject to 50% stretch in the vertical direction, showing the non-homogeneous Green-Lagrange vertical strains (right). For stacked and diamond cells, the ratio between the thickness and the length of the cell walls also remains unchanged, while for staggered and hexagonal cells, this ratio increases slightly.

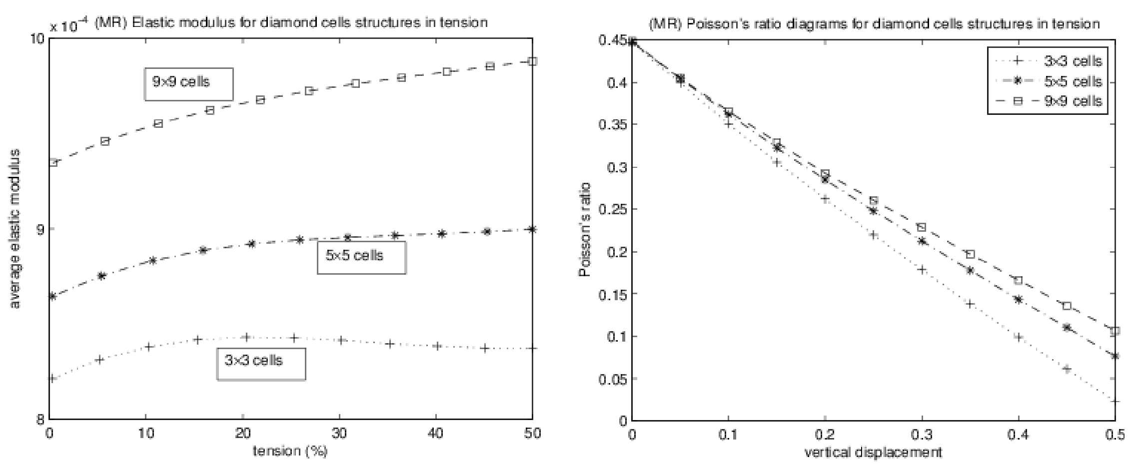

Figure 2. The non-linear elastic modulus (left) and non-linear Poisson's ratio (right) for the diamond cells of Mooney-Rivlin material with the different curves corresponding to structures with a different number of cells while the volume of elastic material is fixed.

CONCLUSION

Thanks to their monotonic behaviour, the non-linear elastic modulus and Poisson's ratio analysed here can be used as indicators for finding the optimum wall thickness or number of cells in similar structures.

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