

Received:  
18 October 2016

Revised:  
30 December 2016

Accepted:  
17 January 2017

Heliyon 3 (2017) e00234



# Tsunami mitigation by resonant triad interaction with acoustic–gravity waves

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## Abstract

Tsunamis have been responsible for the loss of almost a half million lives, widespread long lasting destruction, profound environmental effects, and global financial crisis, within the last two decades. The main tsunami properties that determine the size of impact at the shoreline are its wavelength and amplitude in the ocean. Here, we show that it is in principle possible to reduce the amplitude of a tsunami, and redistribute its energy over a larger space, through forcing it to interact with resonating acoustic–gravity waves. In practice, generating the appropriate acoustic–gravity modes introduces serious challenges due to the high energy required for an effective interaction. However, if the findings are extended to realistic tsunami properties and geometries, we might be able to mitigate tsunamis and so save lives and properties. Moreover, such a mitigation technique would allow for the harnessing of the tsunami's energy.

Keywords: Applied mechanics, Acoustics

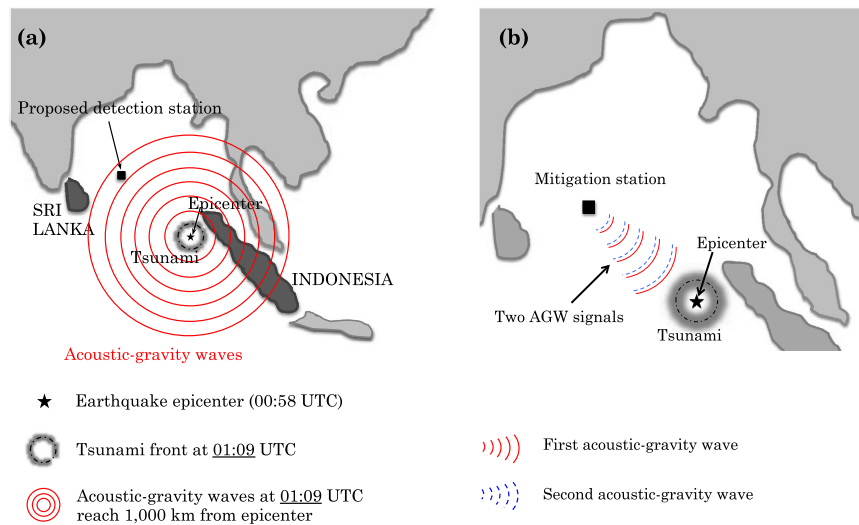
## 1. Introduction

Tsunamis are water waves caused by the displacement of a large volume of water, in the deep ocean or a large lake, following an earthquake, landslide, underwater explosion, meteorite impacts, or other violent geological events. On the coastline, the resulting waves evolve from unnoticeable to devastating, reaching heights of tens

of meters and causing destruction of property and loss of life. Over 225,000 people were killed in the 2004 Indian Ocean tsunami alone. For many decades, scientists have been studying tsunami, and progress has been widely reported in connection with the causes [1], forecasting [2], and recovery [3]. However, to our knowledge, none of the studies ratifies the approach of a direct mitigation of tsunamis, with the exception of mitigation using submarine barriers (e.g. see Ref. [4]). In an attempt to open a discussion on this approach, we examine the feasibility of redistributing the total energy of a very long surface ocean (gravity) wave over a larger space through interaction with acoustic–gravity waves.

Acoustic–gravity waves (AGWs) are sound waves that propagate in the water layer with amplitudes governed by the restoring force of gravity. Since the slight compressibility of the water has a negligible effect on surface gravity waves, on one hand, and the gravitational force has no practical effect on sound waves in the ocean, on the other hand, the compressibility and gravity effects in water have long been treated separately. Consequently, AGWs that rely on the interplay between acoustic and gravity wave modes have not yet received proper attention. AGWs have typically wavelengths of tens or hundreds of kilometers, and propagate at near the speed of sound in water (1500 m/s), at relatively low frequencies ranging from 0.1–100 Hz. Unlike surface ocean waves, AGWs form with tsunamis and induce pressure disturbances not only near the surface but in the whole water column, reaching the sea-floor where they leave measurable pressure signatures [5], which makes them perfect tsunami precursors (Figure 1(a)). In fact, with only two low-frequency bottom-pressure sensors, one can identify the epicenter location, from arrival time delay, and the fact that the pressure decreases proportional to the inverse square-root of the distance — or by employing the inverse solution given by Ref. [6]. Such a detection station should be installed in the deep ocean where AGWs are expected to travel freely in the water column. This has clear benefits relative to standard warning systems that rely on the actual arrival of the tsunami. As an example, the 2004 Indian Ocean earthquake that occurred at 00:58 UTC generated a tsunami that hit the coastal villages of Indonesia 14 minutes later, at 01:12 UCT, and Sri Lanka 71 minutes later at 02:23 UCT. If an AGW based alarm system existed at that time at a distance of a thousand kilometers from the epicenter, almost regardless to direction, the tsunami could have been detected at 01:09 UTC, that is 3 and 60 minutes before it hit Indonesia and Sri Lanka, respectively, which could have saved many lives (see Figure 1(a)).

Besides acting as tsunami precursors, AGWs can exchange and share energy with surface ocean waves in a three-wave interaction mechanism known as a resonant triad (see Refs. [7, 8]). A general theory on resonant triad interactions of AGWs was developed recently [9]. The theory considers the interaction of two surface ocean



**Figure 1.** Detection and mitigation system. (a): Schematic illustration of tsunami and acoustic–gravity waves (AGWs) generated during the 2004 Indian Ocean earthquake. The AGWs travel much faster than the tsunami reaching the proposed detection station, at a distance of 1,000 km from the epicenter, within 11 minutes; that leaves 3 and 60 minutes before the tsunami hits Indonesia and Sri Lanka, respectively. (b): Schematic illustration of the proposed mitigation system. Two AGWs are transmitted towards the tsunami, to form a resonant triad.

wave packets of similar periods but opposite directions and shows how these give rise to an AGW of a similar frequency, but much larger wavelength. It also shows that the interaction of wave packets is far less efficient, in terms of energy exchange, than the interaction of a train of sinusoidal waves. While this energy exchange provides a natural explanation of the generation of oceanic microseisms [10] — small oscillations of the seafloor in the frequency range of 0.1–0.3 Hz — it suggests that mitigation of surface gravity waves is possible through a careful resonant triad interaction.

In order to utilize the suggested mitigation mechanism, we consider a more practical interaction comprising a single long surface ocean wave, representing the tsunami, and two AGWs. In the current settings, all triad members have a comparable lengthscale, whilst the two AGWs have much larger timescales [11]. The wavelength of the tsunami is assumed longer than a regular surface ocean wave, but short enough that the dispersion relation is still observed. Once a tsunami is identified, e.g. using the early detection warning system employed above, we transmit two finely tuned trains of AGWs that upon interaction with the tsunami form a resonant triad, as illustrated in Figure 1(b).

## 2. Background

### 2.1. Governing equations

Following [10], we consider a two dimensional Cartesian coordinate system  $(x, z)$  with the origin in the undisturbed free surface, and the  $z$ -axis vertically upwards; the density is a function of pressure alone, the earth curvature and the viscosity are neglected, and the velocity  $\mathbf{u}$  is assumed irrotational, so that  $\mathbf{u} = \nabla\varphi$ . Let  $z = \eta$  be the equation of the free surface, and  $z = -h$  the equation of the rigid flat bottom. Approximate to quadratic terms, the equations of motion can then be integrated to obtain the field equation [10]

$$\varphi_{tt} - c^2 \nabla^2 \varphi + g\varphi_z = -2\varphi_x \varphi_{xt} - 2\varphi_z \varphi_{zt} \quad (-h \leq z \leq 0), \quad (1)$$

where  $c$  is the speed of sound in the fluid,  $g$  is the gravitational acceleration, and  $t$  is the time.

The boundary condition on the rigid bottom is

$$\varphi_z = 0 \quad (z = -h). \quad (2)$$

On the free surface, the usual kinematic and dynamic conditions apply, and the combined condition read (see Ref. [11])

$$\varphi_{tt} + g\varphi_z = -2\varphi_x \varphi_{xt} - 2\varphi_z \varphi_{zt} + g^{-1} \varphi_t \varphi_{tz} + \varphi_t \varphi_{zz} \quad (z = 0). \quad (3)$$

### 2.2. Dispersion relation

Applying a separation of variables in the linearized field equation results in an ordinary differential equation. The solution of the equation results in the following AGW dispersion relation upon substitution in the boundary conditions,

$$\omega^2 = -g\mu \tan(\lambda h), \quad (4)$$

where  $k^2 = \omega^2/c^2 - \lambda^2$ , with real  $k$  and  $\lambda$ . Note that for a small scale dispersive tsunami of frequency  $\omega = \sigma$ , traveling on deep water,  $\lambda \simeq ik$  is pure imaginary and the dispersion relation reduces to

$$\sigma^2 = gk, \quad (5)$$

which is the well-known surface gravity wave dispersion relation.

### 2.3. Resonant triads

In the problem at hand, the triad comprises two acoustic modes,  $(\omega_1, q_1)$ ,  $(\omega_2, q_2)$ , and a tsunami  $(\sigma, k)$ , that satisfy the resonance conditions

$$\sigma = \omega_1 - \omega_2, \quad k = q_1 + q_2, \tag{6}$$

and the dispersion relations (4) and (5).

Defining a potential that comprises a tsunami and two AGWs, substituting in the governing equations, and imposing a solvability condition, results in the amplitude evolution equations for the tsunami, and the two AGWs in the form

$$\frac{dS}{d\tau} = -\beta A_1^* A_2, \tag{7}$$

$$\frac{\partial A_1}{\partial \tau} = -\gamma_1 \frac{\partial A_1}{\partial \xi} + \alpha_1 A_2 S^*, \quad \frac{\partial A_2}{\partial \tau} = -\gamma_2 \frac{\partial A_2}{\partial \xi} - \alpha_2 A_1 S, \tag{8}$$

where  $\alpha_1$ ,  $\alpha_2$ ,  $\gamma_1$ ,  $\gamma_2$ , and  $\beta$  are constants defined by

$$\alpha_1 = \frac{1}{2g^2 \cos(\lambda_1 h)} \times \left\{ (2g\sigma q_1 + \omega_1^3 + 2\sigma^2 \omega_1 + 2\sigma \omega_1^2 - \sigma^3) \sigma \cos(\lambda_2 h) + 2g\sigma^2 \lambda_2 \sin(\lambda_2 h) \right\}$$

$$\alpha_2 = \frac{1}{2g^2 \cos(\lambda_2 h)} \left\{ (2g\sigma q_1 + \omega_1^3 + \sigma^2 \omega_1 + \sigma \omega_1^2) \sigma \cos(\lambda_1 h) - 2g\sigma^2 \lambda_1 \sin(\lambda_1 h) \right\}$$

$$\gamma_1 = -\frac{gq_1}{\omega_1 \lambda_1 \cos(\lambda_1 h)}, \quad \gamma_2 = -\frac{gq_2}{\omega_2 \lambda_2 \cos(\lambda_2 h)}.$$

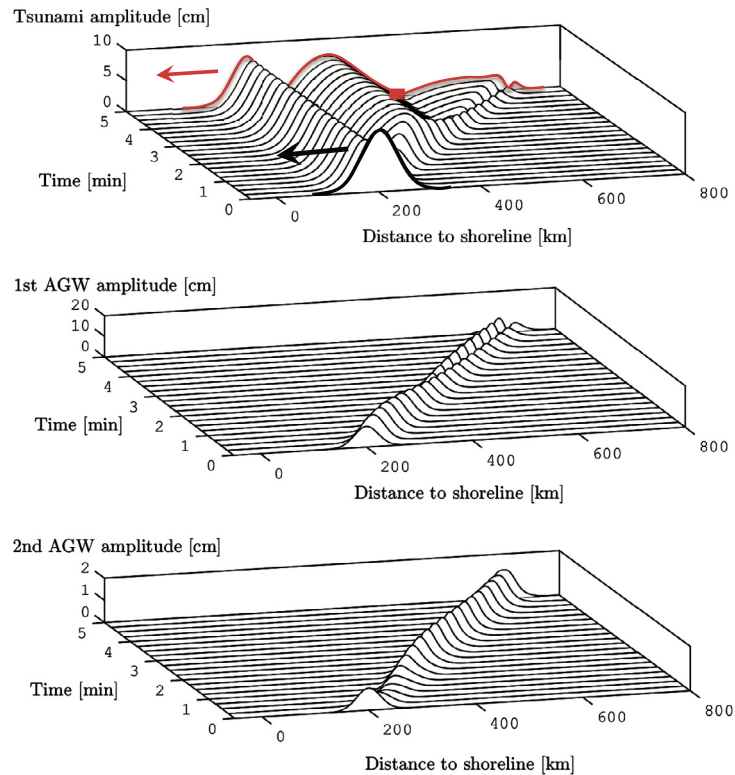
## 3. Results

### 3.1. Redistribution of energy

In order to demonstrate the results effectively we consider a numerical example whereby the depth  $h = 3000$  m,  $c = 1500$  m/s,  $g = 9.81$  m/s<sup>2</sup>;  $\omega_1 = 1$  rad/s, and the corresponding frequencies,  $\sigma = 0.084$  rad/s,  $\omega_2 = 0.916$  rad/s; and  $\alpha_1 = \alpha_2 = 1$ ,  $\beta = -1$ ,  $\gamma_1 = \gamma_2 = 1$ . In addition, we consider the tsunami and the AGW envelopes to be *Gaussian* with initial amplitudes,

$$S_0 = e^{-x^2} \quad A_{10} = e^{-\frac{x^2}{2b^2}}, \quad A_{20} = e^{-\frac{x^2}{2b^2}}, \tag{9}$$

where a standard deviation (Gaussian widths)  $b = 2^{-1/2}$  was considered in the main example. As the interaction proceeds, energy is withdrawn from the tsunami to the AGWs, which due to their high propagation speed transfer the withdrawn energy away from the original tsunami envelope. Thus, the total energy of the tsunami



**Figure 2.** Evolution of amplitudes. As the tsunami propagates from right to left it interacts with two transmitted trains of acoustic–gravity waves, that propagate from left to right. After the interaction, the tsunami envelope is redistributed behind over a larger space and its amplitude is reduced.

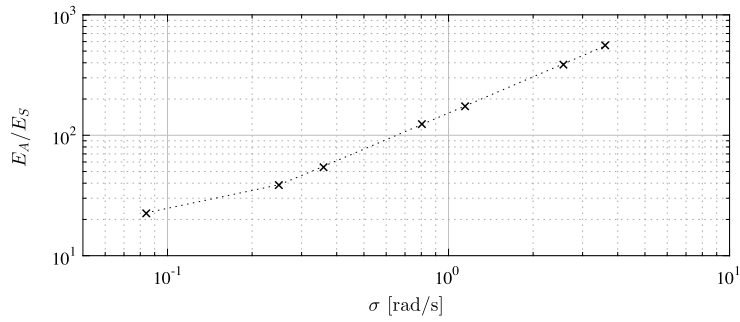
is redistributed over a larger area, and the initial tsunami amplitude is reduced (Figure 2). Consequently, as the tsunami approaches the shoreline, its run-up height decreases accordingly and the impact at the shoreline reduces. In theory, the tsunami energy redistribution process by AGWs can be repeated over and over until the tsunami is completely dispersed, and the run-up height is minimal. However, this may require a very long interaction time in particular for lower frequencies.

### 3.2. Energy estimation

To evaluate the amount of energy within the AGW modes we consider the case without a spatial dependency, whereby an analytical solution in the form of Jacobi elliptic functions can be formulated [11]

$$|S|^2 = |S_0|^2 - \frac{\beta}{\alpha_1} |A_{10}|^2 \text{sn}^2(u, \theta) \tag{10}$$

$$|A_1|^2 = |A_{10}|^2 \text{cn}^2(u, \theta), \quad |A_2|^2 = \frac{\alpha_2}{\alpha_1} |A_{10}|^2 \text{sn}^2(u, \theta) \tag{11}$$



**Figure 3.** Total energy in the two AGW modes.

with argument  $u = \sqrt{\alpha_1\alpha_2}|S_0|\tau$  and modulus  $\theta = |A_{10}|\sqrt{\beta}/(|S_0|\sqrt{\alpha_1})$ . The intrinsic energy of wave action quantities, which can be obtained in the form of *Manley–Rowe* from the evolution equations (without the spatial dependency), take the form [11]

$$E_s = \sigma\beta|S|^2, \quad E_{A1} = \omega_1\alpha_1|A_1|^2, \quad E_{A2} = -\omega_2\alpha_2|A_2|^2 \tag{12}$$

where the total energy  $E_s + E_1 + E_2$  is conserved. The two AGWs have comparable energy, and each AGW has a relative energy of

$$\frac{E_A}{E_S} \sim \frac{\omega\alpha|A|^2}{\sigma\beta|S|^2} = \frac{\omega\alpha}{\sigma\beta} \frac{|A_0|^2\text{sn}^2(u, \theta)}{|S_0|^2 \left(1 - \frac{\beta}{\alpha}|A_{10}|^2\text{sn}^2(u, \theta)\right)} \tag{13}$$

where  $E_A = |E_{A1}| + |E_{A2}|$ . For  $\theta \simeq 1$  eq. (13) reduces to form

$$\frac{E_A}{E_S} \sim \frac{\omega|A_0|^4}{\sigma|S_0|^4} \tag{14}$$

Since  $\sigma = \epsilon\omega$ , where  $\epsilon \ll 1$ ,  $(E_A/E_S) \sim O(1/\epsilon)$  when  $A_0$  and  $S_0$  are comparable; and  $(E_A/E_S) \sim O(\epsilon)$  when  $(A_0/S_0) \sim O(\epsilon^{1/2})$ .

Note that the energy required for the AGWs increases almost linearly with the tsunami frequency as depicted in Figure 3. For a tsunami with  $\sigma = 0.25$  rad/s, and a total energy in the order of Tera Joules, the total energy of the required AGW triad counterparts is comparable to the energy in ‘The Little Boy’ atomic bomb dropped on Hiroshima on August 6, 1945.

### 3.3. Interaction efficiency

In order to achieve an efficient resonance, the two AGW trains have to be finely tuned, and their amplitudes carefully chosen. If all three waves were standing, i.e. not propagating, then the energy exchange would be transferring between the triad members in a cyclic way, as in a classical triad resonance [7]. However, the interaction here results in a redistribution of energy for each of the propagating

**Table 1.** AGW envelope width and the associated energy.

Standard deviation $b$	Amplitude reduction $\Delta S$	Relative energy $E_A/E_S$
0.707	37%	25
0.223	8%	8
0.158	5%	5
0.111	3%	4

waves. If initially we allow only a single AGW to interact with the tsunami, then a second AGW would be generated, though most of the energy would transfer between the two AGWs leaving the tsunami almost completely unaltered [11]. Therefore, in order to guarantee a significantly larger amount of energy withdrawal from the tsunami, we prescribe the amplitudes of the two AGWs, comparably, at the initial stage. Now, as the triad interaction develops, the amplitudes of all three mode envelopes attenuate (Figure 2). Since the AGWs travel much faster than the tsunami, and in an opposite direction (from left to right), they transfer part of the withdrawn energy outside of the original gravity mode envelope, resulting in a redistribution of the latter's energy both in time and space. Thus, the two AGW trains form a secondary gravity wave envelope, then a tertiary, and so on until no further interaction is possible. In the example given, the original amplitude of the gravity envelope drops by almost 30%, and the secondary envelopes are positioned behind, i.e. their arrival to the shoreline is delayed. The corresponding run-up height is reduced by approximately 17%, e.g. in the 2004 Indian Ocean tsunami this would have resulted in a decrease of at least 5 meters in the tsunami run-up height. Such a fractional reduction of the amplitude or an increase of the delay could have saved many lives.

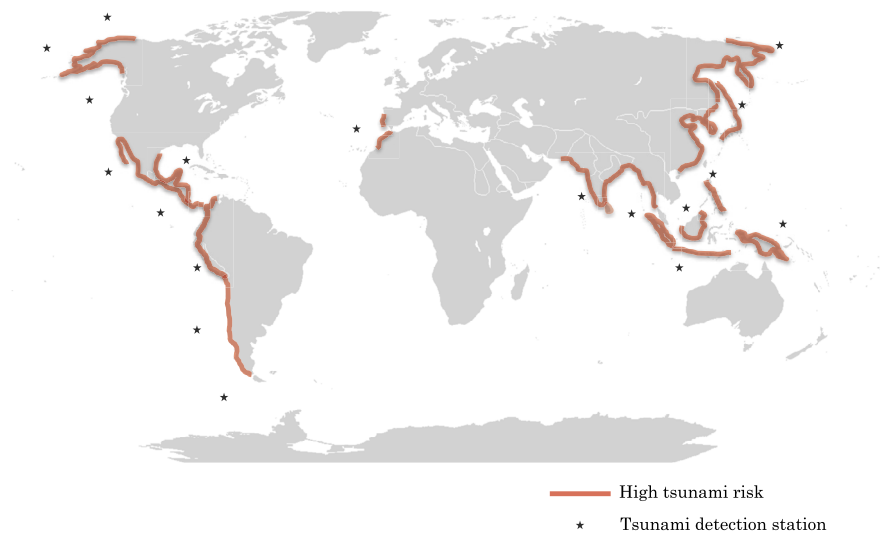
The energy in AGWs can be further optimized by tuning the standard deviation  $b$  in eq. (9). For  $b = 0.707$  there is a reduction of 37% in the amplitude of the primary tsunami envelope, which requires energy input 25 times that in the tsunami. Considering much narrower AGW-packets, say by taking  $b = 0.111$ , reduces the energy input by a factor of 6. However, the amplitude of the tsunami envelope drops only by 3%. Table 1 summarizes the impact of different values of  $b$ .

#### 4. Discussion

The amount of energy required to generate AGWs, given a realistic scenario, is probably much higher than the AGW energy, whereas the associated amplitude reduction is probably far less efficient. Thus, there is a need to improve the interaction efficiency further. This might be achieved by considering higher AGW modes, which I have not discussed here. With higher AGW modes, one anticipates not only the



## Worldwide tsunami early detection system



**Figure 4.** Schematic representation of high tsunami risk areas and potential distribution of detection stations that would allow early alarm even in case of tsunami generation near the shoreline, as in the 2004 Indian Ocean tsunami case.

input energy to be lower, but the interaction timescale would become much shorter, enabling multiple interactions during the same time period.

Although the technical aspects of the generation of AGWs have not been addressed in this work, it is worth mentioning that if AGWs are generated mechanically then one expects the lengthscale of the mechanics involved to be comparable to the lengthscale of the AGWs, hence impractically long. An alternative could be the use of naturally generated AGWs by the same earthquake, which need to be modulated to meet the resonance conditions.

The tsunami early detection and mitigation mechanisms presented here are appropriate for gravity waves with periods reaching a few minutes at most caused by localized tectonic movements, or non-seismic sources, such as submarine mass failures [12, 13]. For larger-scale tsunamis, one should account for the interaction of a non-dispersive gravity wave, with two dispersive AGWs. Resonant triads involving a non-dispersive mode have been studied in the past [14], though here the interaction involves fast and slow waves, rather than short and long. One could adapt the mechanisms presented here to account for other violent geophysical processes in the ocean such as landslides, volcanic eruptions, underwater explosions, and falling meteorites. While the scales involved may differ in each process, the underlying physical processes involved are similar.

It is also noteworthy that installing an early tsunami detection system is feasible and basically requires installation of a standard low frequency “off-the-shelf”

hydrophones system in the deep ocean. As a final note, even if we consider extreme tsunami scenarios, in terms of proximity to shoreline as in the 2004 tsunami case, not many AGW-detection stations would be required to provide a worldwide alarm system that would serve all tsunami high risk areas (Figure 4). While detection is relatively straight forward, the mitigation of tsunamis requires the design of highly accurate AGW frequency transmitters or modulators, which is a rather challenging and ongoing engineering problem.

## Declarations

### Author contribution statement

Kadri Usama: Conceived and designed the analysis; Analyzed and interpreted the data; Contributed analysis tools or data; Wrote the paper.

### Funding statement

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

### Competing interest statement

The authors declare no conflict of interest.

### Additional information

No additional information is available for this paper.

### Acknowledgements

I'm very grateful to Walter Munk, John Bush, and Victor Shrira for useful discussions, and to the referees for their constructive comments and recommendations.

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