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Optimal Configuration of Soft Open Point for Active Distribution Network Based on Mixed-integer Second-order Cone Programming

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Abstract

Soft Open Point (SOP) refers to a power electronic device installed in place of a normally-open point in a distribution network. The application of SOP will greatly promote the economy, flexibility and controllability of the distribution network. In this paper, an optimal configuration method of SOP is proposed for the operation of active distribution system. Firstly, considering the characteristics of renewable energy generation, classic scenarios are constructed based on Wasserstein distance metric. Secondly, an optimal configuration model of SOP is presented. Then, a conic model conversion is proposed and mixed-integer second-order cone programming is used to solve the model with efficiency and convergence. Finally, case studies on IEEE 33-node test feeder are used to verify the proposed method.

Keywords: distributed generation (DG); soft open point (SOP); optimal configuration; Wasserstein distance metric; mixed-integer second-order cone programming (MISOCP)

1. Introduction

With the increasing integration of distributed generation (DG), the distribution system will give full expression to these aspects, including loss reduction, power supply reliability improvement, economics promotion and environmental pollution advancement, *etc*. However, the widely application of DGs, especially the intermittent DGs pose new challenges to the operating conditions, such as voltage exceeding,

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network congestion and the randomness of DG power supply [1]. Traditional distribution systems are incompetent to deal with the large integration of intermittent DGs due to their limited adjusting means. Soft open point (SOP), a new type of intelligent power distribution device, derives under the above background to replace normally-open point (NOP). Compared with NOP, the power control of SOP is more safe and reliable, and even able to realize the real-time optimization control. SOP is mainly based on the fully-controlled power electronic devices, which leads to higher investment and operation maintenance cost. Therefore, it is very necessary to make a reasonable configuration scheme for SOP. Reference [2] studied the basic principle and model of the SOP, while reference [3, 4] carried on the simulation analysis of steady-state and transient characteristics of SOP, respectively, which will provide a foundation for the optimal configuration of SOP.

An optimal configuration model of SOP considering DG operation characteristics is proposed in this paper. The model is essentially a mixed-integer nonlinear problem. The optimal scenario generation technique based on Wasserstein distance is adopted to build classic scenarios for the problem to be solved [5]. Furthermore, a conic model conversion is proposed to realize the required format, and the mixed-integer second-order cone programming (MISOCP) is used to solve the model. Finally, the optimal configuration model of SOP and MISOCP are verified on the IEEE 33-node test feeder.

2. Optimal configuration modelling of SOP in active distribution network

2.1. Scenario generation based on Wasserstein distance metric

The uncertainty of DG is usually represented by a continuous probability density distribution function. In the modelling stage, the discrete distribution instead of the continuous distribution is used to simplify the model, which is also called scenario generation. Taking into account that the power flow optimization problem is complex, it is difficult to approximate the original probability density distribution function with less discrete scenarios. In the scenario generation technique, the method based on Wasserstein distance metric is adopted in this paper.

Assuming that continuous probability density function of variable x is $f(x)$, S discrete scenarios are used to approximate $f(x)$. The optimal scenario z_s (s = 1, 2, ···, S) can be obtained by the formula (1).

$$\int_{-\infty}^{z_s} f(x)^{\frac{1}{r+1}} dx = \frac{2s-1}{2S} \int_{-\infty}^{+\infty} f(x)^{\frac{1}{r+1}} dx \tag{1}$$

2.2. Optimal configuration model

In this paper, we take the minimum annual expense of overall distribution system as the objective function, which is formulated as

$$\min C = C^{\text{INV}} + C^{\text{OPE}} + C^{\text{LOSS}} \tag{2}$$

The annual expense of overall distribution system is composed of the following three parts.

1) C^{INV}: fixed investment cost of SOP

$$C^{\text{INV}} = \frac{d(1+d)^y}{(1+d)^y - 1} \sum_{i=1}^{N} \sum_{j\in\Omega(i)} c^{\text{SOP}} S_{ij}^{\text{SOP}} \tag{3}$$

Where d is the discount rate and y is the SOP limited lifetime. Let N denote the number of all buses and $\Omega(i)$ be the set of all adjacent nodes of bus i. c^{SOP} denotes the investment cost of unit capacity and S_{ij}^{SOP} denotes the capacity of SOP installed between node i, j.

2) C^{OPE}: annual operation cost of SOP

$$C^{\text{OPE}} = \eta \sum_{i=1}^{N} \sum_{j\in\Omega(i)} c^{\text{SOP}} S_{ij}^{\text{SOP}} \tag{4}$$

Where η is the coefficient of annual operation cost.

3) C^{LOSS}: annual cost of losses in distribution system

$$C^{\mathrm{LOSS}} = 8760c\left(\sum_{s=1}^{S}\sum_{i=1}^{N}\sum_{j\in\Omega(i)} r_{ij} I_{s,ij}^2 + \sum_{s=1}^{S}\sum_{i=1}^{N} A_i^{\mathrm{SOP}}\left|P_{s,i}^{\mathrm{SOP}}\right|\right) \tag{5}$$

Let s denote the index of scenario and S denote the set of all scenarios. Let c denote the electricity price. Denote $z_{ij} = r_{ij} + \mathbf{i}x_{ij}$ as the complex impedance of the line connecting bus i and bus j. Let $I_{s,ij}$ denote the current from bus i to bus j in sth scenario; $P_{s,i}^{\mathrm{SOP}}$ is the transmitted active power of SOP at node i in sth scenario and A_i^{SOP} is the loss coefficient at node i.

1) Radical distribution system constraints

$$\alpha_{ij} = \beta_{ij} + \beta_{ji} \tag{6}$$

$$\sum_{j\in\Omega(i)} \beta_{ij} = 1, \quad \forall i \in N \setminus N^{\mathrm{S}} \tag{7}$$

$$\sum_{j\in\Omega(i)} \beta_{ij} = 0, \quad \forall i \in N^{\mathrm{S}} \tag{8}$$

Let N^{S} denote the subset of buses which are substations. Let a single binary variable α_{ij} designate the switch status of the line. For example, $\alpha_{ij} = 1$ implies that the switch of line l is closed, $\alpha_{ij} = 0$ indicates open conversely. Since the binary variable α_{ij} is not directional, the summation of α_{ij} over all lines is exactly the number of closed switches. Each switch status of the line is also associated with two continuous line orientation variables β_{ij} and β_{ji}, representing the direction of power flow.

2) System power flow constraints

$$\sum_{j\in\Phi(i)} \alpha_{ji}\left(P_{s,ji} - r_{ji} I_{s,ji}^2\right) + P_{s,i} = \sum_{k\in\Psi(i)} \alpha_{ik} P_{s,ik} \tag{9}$$

$$\sum_{j\in\Phi(i)} \alpha_{ji}\left(Q_{s,ji} - x_{ji} I_{s,ji}^2\right) + Q_{s,i} = \sum_{k\in\Psi(i)} \alpha_{ik} Q_{s,ik} \tag{10}$$

$$P_{s,i} = P_{s,i}^{\mathrm{DG}} + P_{s,i}^{\mathrm{SOP}} - P_{s,i}^{\mathrm{LD}} \tag{11}$$

$$Q_{s,i} = Q_{s,i}^{\mathrm{DG}} + Q_{s,i}^{\mathrm{SOP}} - Q_{s,i}^{\mathrm{LD}} \tag{12}$$

$$I_{s,ij}^2 = \frac{P_{s,ij}^2 + Q_{s,ij}^2}{U_{s,i}^2} \tag{13}$$

Equations (9) and (10) represent the law of conservation of power at each bus i in sth scenario, where $P_{s,i}$ and $Q_{s,i}$ denote the real and reactive power injection at bus i, respectively. Let $\Phi(i)\subseteq N$ be the set of all parents of bus i and $\Psi(i)\subseteq N$ be the set of all children of bus i. Equations (11) and (12) indicate the power injection at each bus i, which is the generation minus the load on bus i in sth scenario. Let $P_{s,i}^{\mathrm{DG}}$ and $Q_{s,i}^{\mathrm{DG}}$ represent the real and reactive power generated by DG that are connected to the bus i via inverters, respectively. And let $P_{s,i}^{\mathrm{LD}}$ and $Q_{s,i}^{\mathrm{LD}}$ be the real and reactive power consumption in sth scenario, respectively. Let $P_{s,i}^{\mathrm{SOP}}$ and $Q_{s,i}^{\mathrm{SOP}}$ be the real and reactive power delivered by SOP. The current magnitude of each line can be determined by using Equation (5), where $U_{s,i}$ denotes the complex voltage of bus i in sth scenario. Equation (13) represents Ohm's law over each link (i, j).

3) System operation constraints

$$\left(U_i^{\min}\right)^2 \le U_{s,i}^2 \le \left(U_i^{\max}\right)^2 \tag{14}$$

$$0 \le I_{s,ij}^2 \le \left(I_{ij}^{\max}\right)^2 \tag{15}$$

$U_i^{\max}$ and $U_i^{\min}$ are the upper and lower limits of voltage amplitude at node i, respectively. Let $I_{ij}^{\max}$ denote the upper limit of current amplitude of line ij.

4) SOP operation constraints

$$P_{s,i}^{\mathrm{SOP}} + P_{s,j}^{\mathrm{SOP}} + A_i^{\mathrm{SOP}}\left|P_{s,i}^{\mathrm{SOP}}\right| + A_j^{\mathrm{SOP}}\left|P_{s,j}^{\mathrm{SOP}}\right| = 0 \tag{16}$$

$$-\mu S_{ij}^{\mathrm{SOP}} \le Q_{s,i}^{\mathrm{SOP}} \le \mu S_{ij}^{\mathrm{SOP}}, \quad -\mu S_{ij}^{\mathrm{SOP}} \le Q_{s,j}^{\mathrm{SOP}} \le \mu S_{ij}^{\mathrm{SOP}} \tag{17}$$

$$\sqrt{\left(P_{s,i}^{\mathrm{SOP}}\right)^2+\left(Q_{s,i}^{\mathrm{SOP}}\right)^2} \le S_{ij}^{\mathrm{SOP}},\ \sqrt{\left(P_{s,j}^{\mathrm{SOP}}\right)^2+\left(Q_{s,j}^{\mathrm{SOP}}\right)^2} \le S_{ij}^{\mathrm{SOP}} \tag{18}$$

Let μ denote the absolute value of the sine of power factor angle.

SOP is installed in place of NOP in distribution network, assuming that its allocation capacity is a discrete variable .Thus the relationship between location and capacity of SOP is represented as follows.

$$S_{ij}^{\mathrm{SOP}} = m_{ij} s^{\mathrm{SOP}} \left(1-\alpha_{ij}\right) \tag{19}$$

Where s^{SOP} is the unit installed capacity of SOP. m_{ij} is the number of sets of SOP.

The variables in above-mentioned optimal configuration model include the installing location and capacity of SOP, the switch state and the transmitted active and reactive power of SOP in each scenario. As a consequence, Equations (1)-(19) form the optimization model of optimal configuration of SOP in active distribution system.

2.3. Transformation to mixed-integer second-order cone model

Second-order cone programming (SOCP) is essentially convex programming mathematically, which can be regarded as the generalization of both linear and nonlinear programming. SOCP can solve the problem of minimum linear objective function based on convex cone in the linear space. It has an excellent performance of global optimality and computational efficiency. The SOCP standard form can be written as [6]:

$$\min\{\boldsymbol{c}^T \boldsymbol{x} \mid \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}, \boldsymbol{x} \in \boldsymbol{K}\} \tag{20}$$

As is shown above, the second-order cone programming has very strict demands on the mathematical formulation. The objective function must be a linear function of decision variable $\boldsymbol{x}$ and its feasible region is composed of linear equality constraints and convex cone constraints. The above-mentioned optimization model is described with numerous nonlinear functions, such as the square representation of voltage amplitude and current amplitude. Firstly, it needs to introduce additional variables for each bus and line, respectively, to realize the linearization of nonlinear functions by means of variable substitution.

For each bus $i, j \in N$, let $\tilde{U}_{s,i}$ denote the square of the magnitude of its complex voltage, and let $\tilde{I}_{s,ij}$ denote the square of the magnitude of the complex current. Then, substituting the new optimization variables $\tilde{U}_{s,i}$ and $\tilde{I}_{s,ij}$ into the branch flow model, the objective function and nonlinear constraints become linear except the equality constraints (13). To cast them as second-order cone constraints, these nonlinear equality constraints are relaxed to the inequality constraints. The corresponding SOCP formulation is

$$\left\| 2P_{s,ij} \quad 2Q_{s,ij} \quad \tilde{I}_{s,ij} - \tilde{U}_{s,i} \right\|_2 \le \tilde{I}_{s,ij} + \tilde{U}_{s,i} \tag{21}$$

Equation (21) can make the decision variables constitute the quadratic cone form perfectly, and limit the search space within the convex cone range at the same time.

As for the capacity constraints of SOP, they are transformed into the following form in accordance with requirements of SOCP.

$$\left(P_{s,i}^{\mathrm{SOP}}\right)^2+\left(Q_{s,i}^{\mathrm{SOP}}\right)^2 \le 2*\frac{S_{ij}^{\mathrm{SOP}}}{\sqrt{2}}*\frac{S_{ij}^{\mathrm{SOP}}}{\sqrt{2}},\ \left(P_{s,j}^{\mathrm{SOP}}\right)^2+\left(Q_{s,j}^{\mathrm{SOP}}\right)^2 \le 2*\frac{S_{ij}^{\mathrm{SOP}}}{\sqrt{2}}*\frac{S_{ij}^{\mathrm{SOP}}}{\sqrt{2}} \tag{22}$$

Equation (22) is equivalent to the original constraint (17) while meeting the requirement of rotated quadratic cone form.

As for the absolute terms $\left|P_{s,i}^{\mathrm{SOP}}\right|$ and $\left|P_{s,j}^{\mathrm{SOP}}\right|$ in (2) and (15), auxiliary variables $\left|M_{s,i}^{\mathrm{SOP}}\right|$ and $\left|M_{s,j}^{\mathrm{SOP}}\right|$ are introduced to represent and linearize them, as shown in follows.

$$M_{s,i}^{\mathrm{SOP}} \ge 0,\quad M_{s,j}^{\mathrm{SOP}} \ge 0 \tag{23}$$

$$M_{s,i}^{\mathrm{SOP}} \ge P_{s,i}^{\mathrm{SOP}},\ M_{s,i}^{\mathrm{SOP}} \ge -P_{s,i}^{\mathrm{SOP}} \tag{24}$$

$$M_{s,j}^{\mathrm{SOP}} \ge P_{s,j}^{\mathrm{SOP}},\ M_{s,j}^{\mathrm{SOP}} \ge -P_{s,j}^{\mathrm{SOP}} \tag{25}$$

After conic conversion, the MISOCP model for the optimal configuration of SOP is built.

3. Case study

In this section, the effectiveness of the mentioned mixed-integer second-order cone model is validated on the IEEE 33-node test feeder. The voltage level is 12.66kV and it has 37 branches with 5 tie switches, as shown in Fig.1. Total active power consumption is 3635kW and reactive power consumption is 2265kvar. Three wind turbines of 200/200/300 kVA and two photovoltaic systems of 300/400 kVA are integrated at nodes 17, 30, 33, 13 and 27, respectively. The main parameters are shown in Table 1.

The MISOCP model is implemented in the optimization toolbox YALMIP, and solved with IBM ILOG CPLEX Optimization Studio 12.5 using default parameter settings. The computation is carried out on a PC with Intel Xeon CPU E5-1620 @3.70GHz and a 32GB RAM.

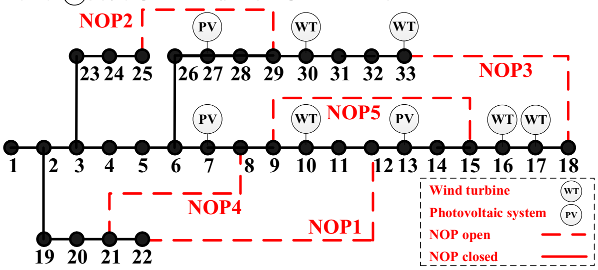

Fig. 1. Structure of the IEEE 33-node test feeder

Table 1. Parameters of the studied case

Parameters	Discount rate	Economical service life of SOP	Minimum optimum capacity of SOP	Unit capacity investment of SOP	Loss coefficient of SOP
value	0.08	20 years	100 kVA	$152 / kVA	0.0199

Based on the Wasserstein distance metric optimal scenario generation technology, five wind power scenarios, five photovoltaic scenarios and their corresponding probability are obtained simultaneously. Then the combined 25 scenarios and the corresponding probability can be also derived. On that basis, the following two schemes are adopted to make the location and capacity optimal configuration of SOP.

Scheme I: SOPs are installed only instead of normally open points;

Scheme II: SOPs are installed instead of normally open points and normally closed points.

The optimal configuration scheme is shown in Table 2, and the results are shown in Table 3.

Table 2. Location and sizing of SOP

Scheme I	Location	12-22	25-29	18-33	8-21	9-15
	capacity / kVA	--	--	200	100	300
Scheme II	Location	6-7	9-10	14-15	31-32	25-29
	capacity / kVA	--	--	--	300	--

Table 3. Optimal configuration results

	C^{INV}	C^{OPE}	C^{LOSS}	C
Without SOP	--	--	$68,111.2	$68,111.2
Scheme I	$11,111.2	$912.0	$51,588.8	$63,612.0
Scheme II	$5,548.0	$456.0	$47,256.8	$53,260.8

It can be observed from Table 3 that Scheme II has better economic benefits. The annual total cost of Scheme II is $14,850.4 less than that without SOP, decreased 21.8 percent than before. Among all expenses, distribution system loss cost is reduced by $20,854.4 with a 30.62 percent reduction, so that the economic operation of the power distribution system is greatly improved.

In order to verify the efficiency and convergence of MISOCP for solving the SOP programming problem and test the correctness of transformed cone model, we take KNITRO and BONMIN in GAMS package as a comparison. With the same example parameters for optimization, the quantity of continuous variables in each scheme is 4750 considering the 25 scenarios. In Scheme I, all the algorithms can converge and get the same SOP optimal configuration result with 5 integer variables, while in Scheme II KNITRO and BONMIN can not converge with 42 integer variables. Table 4 gives the solving time of different algorithms.

Table 4. Comparison of the performance with MISOCP, KNITRO and BONMIN

	MISOCP	KNITRO	BONMIN
Scheme I	13.92s	68.38s	720.01s
Scheme II	2167.27s	divergent	divergent

It can be seen from Table 4, when integer variables are less, all the algorithms can converge to the same optimizing result, and MISOCP can solve the problem efficiently. When the quantity of integer variables is increasing, the rapid growth in the scale of problem will aggravate the divergence of KNITRO and BONMIN. Due to linearization and convex relaxation of the original problem, MISOCP has shown good convergence by reducing the difficulty of the problem. Therefore, the MISOCP used in this paper is more suitable for solving this problem than some commercial software algorithms.

4. Conclusion

In this paper, an optimal configuration model of SOP is proposed, which considers the distributed power operation characteristics. By the optimal scenario generation based on the Wasserstein distance, MISOCP is used to solve the above problem. After linearization and convex relaxation of the model, efficiency and convergence are improved for the optimization algorithm. The reasonable optimal configuration of SOP can significantly improve the economic operation of distribution system. With the development of renewable energy integration, flexible control mode of SOP will also bring many benefits to the entire distribution system, which has a good application prospect.

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