Robust solutions to the pollution-routing problem with demand and travel time uncertainty

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Abstract

Freight transportation activities could potentially lead to detrimental effects on the natural and built environments and pose health risks. The importance of the present study is to consider demand and travel time uncertainty in green transport planning by proposing several robust optimization techniques; soft worst case, hard worst case and chance constraints. These techniques provide the most reliable solutions with very limited increase in the objective function related to fuel consumption and CO_2-equivalent emissions.

Keywords: Green vehicle routing, Freight transportation, Robust optimization, CO_2e emissions

1. Introduction

Green vehicle routing is related to dispatching goods not only based on economic goals, but also by considering the relevant harmful impacts on the environment. Transportation has detrimental effects on the environment such as resource depletion, land use, acidification, toxic effects on ecosystems and humans, noise and the impacts induced by Greenhouse Gas (GHG) emissions (Knörr, 2009). GHGs absorb and emit radiations within the thermal infra-red range in the atmosphere and significantly raise the Earth's temperature. As of August 2016, the level of atmospheric CO_2 emissions is estimated to be equal to 402.25 ppm and is still increasing (ESRL, 2016). The emissions of CO_2 are directly proportional to the amount of fuel consumed by a vehicle, which is in turn dependent on a variety of vehicle, environment and traffic-related parameters, such as vehicle speed, load and road gradient (Demir et al., 2011, 2014). The carbon dioxide equivalent (CO_2e) measures how much global warming a given type and amount of GHG may cause, using the functionally equivalent amount or concentration of CO_2 as the reference.

The Vehicle Routing Problem (VRP) is a well-known NP-hard problem which was introduced by Dantzig and Ramser (1959). Since then, VRP has been a topic of numerous studies in the

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literature of operations research. The traditional VRP includes a set of costumers with known demands, a single depot, and a homogeneous fleet of vehicles by determining a set of routes. The literature on VRP and its variants are very widespread and involve many different aspects and decisions (see, for example, the latest surveys of Golden et al. (2008); Eksioglu et al. (2009); De Jaegere et al. (2014)). Also various exact (see, e.g., Baldacci et al., 2012; Almoustafa et al., 2013) and heuristics algorithms (see, e.g., Demir et al., 2012; Kramer et al., 2015) are suggested to solve such operational-level routing problems.

The traditional objective in the standard VRP is to minimize the total distance traveled by all vehicles, but this objective can be enriched through the inclusion of terms related to fuel consumption (Bektaş and Laporte, 2011). Recent developments in Green Vehicle Routing Problems (GVRPs) have heightened the importance of operations research techniques in this area. One of the successful applications in GVRP is due to Bektaş and Laporte (2011) who introduced the Pollution-Routing Problem (PRP), which is an extension of the VRP with time windows (VRPTW). In this paper, we consider a special case of the PRP where the objective function solely depends on the total fuel consumption rather than a weighted sum of fuel consumption and total driving time as in the PRP.

Demand uncertainty is one of the most common variants in non-deterministic (stochastic) VRPs. However, to the best of our knowledge, it has not been studied in the domain of GVRPs. Travel time uncertainty has been also investigated to see the impact of congested travel speeds on fuel consumption. The contributions of the paper are two-fold: (*i*) we investigate the previous robust VRP models with stochastic demand and illustrate their weakness for the PRP, and (*ii*) we reformulate the PRP with several well-known robust approaches.

The remainder of this paper is organized as follows. In Section 2, literature review on both VRPs and GVRPs is provided. In section 3, the mathematical formulation of the PRP is presented. Section 4 introduces the proposed robust optimization approaches along with their formulations. Extensive numerical experiments and comparative analysis are provided in Section 5 to show the powerfulness of the proposed robust optimization models. Finally, Section 6 concludes the paper and introduces relevant future research directions.

2. Literature review

This section reviews the existing research literature on both VRP with uncertain data and green vehicle routing problems.

2.1. VRP with uncertain data

Nondeterministic VRP refers to situations that all information about the vehicle routing is not deterministic before the start of the planning, and some of the information may be uncertain, ambiguous, or even unknown (Chen et al., 2013). In the real world applications the input data of VRP is highly tainted with uncertainty. This has triggered the development of several techniques to handle the imprecision of uncertain data in VRPs. In the literature, VRP with uncertain data has been studied in four groups; namely *stochastic VRP*, *dynamic VRP*, *fuzzy VRP*, and *robust VRP*.

Stochastic VRP (SVRP) has been extensively applied in the condition that the probability distribution of the uncertain parameter is available according to sufficient and reliable historical data (see, e.g., Li et al., 2010; Faulin et al., 2011). Generic variants of SVRP are VRP with stochastic customer demands (see, e.g., Novoa et al., 2006; Smith et al., 2010), VRP with stochastic customers (Bertsimas and Van Ryzin, 1991), VRP with stochastic customers and demands (Gendreau et al., 1995), and VRP with stochastic service times (Laporte, 1992). Interested readers are referred to Berhan et al. (2014) for a survey on different types of SVRP and its solution methodologies. The first study on *dynamic VRP* (DVRP) is done by Wilson and Colvin (1977). The authors investigated a single vehicle dynamic arc routing problem which involves dynamic demand. We refer to Pillac et al. (2013) for the DVRP and its solution methods. Fuzzy number, or *Fuzzy VRP* (FVRP), is one of the most common approaches for modelling optimization problems which have one or more uncertain data with vagueness or ambiguity. Chen et al. (2013) categorized related works in three groups; namely VRP with fuzzy demands (Chen et al., 2006; Peng and Qian, 2010), VRP with fuzzy due times (see, e.g., Zhang et al., 2008; Jun, 2009) and VRP with fuzzy travel times (Brito et al., 2010).

Robust VRP (RVRP) was introduced by Bertsimas and Simchi-Levi (1996) for the SVRP and is applied when the probability distribution of uncertain parameter is unknown (i.e., deep uncertainty). Sungur et al. (2008) introduced a robust optimization approach to solve the capacitated VRP with demand uncertainty and compared the performance of robust solutions with deterministic ones. In a related work, Lee et al. (2012) addressed the VRP with deadlines which involves demand and travel time uncertainty. Adulyasak and Jaillet (2014) described models and algorithms for SVRP and RVRP with deadlines. The authors proposed new mathematical formulations to solve these problems based on a branch-and-cut framework. Solano-Charris et al. (2014) studied the RVRP with uncertain traveling costs. The authors proposed a heuristic approach to solve the investigated problem. The summary of the current studies are listed in Table 1.

Table 1
Recent studies on VRP with uncertain data

Reference	**Uncertain parameters**				Type of VRP	Modeling approach
	Demand	Traveling time/cost	Costumer	Service or due time		
Novoa et al. (2006)	✔				CVRP	SVRP
Chen et al. (2006)	✔				CVRP	FVRP
Sungur et al. (2008)	✔				CVRP	RVRP
Zhang et al. (2008)				✔	VRPTW	FVRP
Jun (2009)				✔	VRPTW	FVRP
Brito et al. (2010)		✔			VRPTW	FVRP
Peng and Qian (2010)	✔				CVRP	FVRP
Smith et al. (2010)	✔				CVRP	SVRP
Li et al. (2010)		✔		✔	VRPTW	SVRP
Faulin et al. (2011)	✔				CVRP	SVRP
Lee et al. (2012)	✔	✔			VRPTW	RVRP
Agra et al. (2013)		✔			VRPTW	RVRP
Adulyasak and Jaillet (2014)		✔			VRPD	RVRP/SVRP
Solano-Charris et al. (2014)		✔			CVRP	RVRP

VRPTW: Vehicle Routing Problem with Time Windows; CVRP: Capacitated Vehicle Routing Problem; VRPD: Vehicle Routing Problem with Deadline; RVRP: Robust Vehicle Routing Problem; FVRP: Fuzzy Vehicle Routing Problem; DVRP: Dynamic Vehicle Routing Problem; SVRP: Stochastic Vehicle Routing Problem

2.2. Green vehicle routing problems

Green vehicle routing is a branch of green logistics which refers to vehicle routing problems where negative externalities, such as CO_2e emissions, are explicitly taken into account so that they are reduced through better planning (Demir et al., 2014). One of the successful applications of the GVRP is the Pollution-Routing Problem, which is coined by Bektaş and Laporte (2011). The PRP is an extension of the classical VRPTW, where the aim is to route a number of vehicles to serve a set of customers and to determine their speed on each route segment, so as to minimize a function comprising fuel, emissions and driver costs. The interested readers are referred to Demir et al. (2014) for a literature survey on GVRPs. A summary of the GVRP studies is given in Table 2.

Table 2
Recent studies on GVRPs

Reference	CO_2 or CO_2e emissions	Type of VRP	Modeling approach
Bektaş and Laporte (2011)	✔	PRP	D
Faulin et al. (2011)	✔	CVRP	D
Suzuki (2011)	✔	TSPTW	D
Ubeda et al. (2011)	✔	VRPCB	D
Xiao et al. (2012)	✔	CVRP	D
Demir et al. (2012)	✔	PRP	D
Jemai et al. (2012)	✔	CVRP	D
Franceschetti et al. (2013)	✔	TDPRP	D
Demir et al. (2013)	✔	BiPRP	D
Kopfer et al. (2014)	✔	FVRPTW	D
Pradenas et al. (2013)	✔	VRPBTW	D
Koç et al. (2014)	✔	MFPRP	D
Zhang et al. (2014)	✔	CVRP	D
Kramer et al. (2015)	✔	PRP	D
Dabia et al. (2016)	✔	PRP	D

TDVRP: Time-Dependent Vehicle Routing Problem; VRPCB: Vehicle Routing Problem with Clustered Backhauls; D: Deterministic

Despite the fact that the number of studies on GVRPs is increasing in recent years, current studies are still restricted and have not covered many practical aspects (e.g., data uncertainty). Because of the cumulative variables in GVRPs, previous models and algorithms used for traditional variants of the VRP cannot be directly applied in our study. To overcome the literature gap, this paper addresses the uncertain customers' demands and travel times. In order to provide an appropriate decision making tool for less conservative decision makers, several flexible robust optimization models are proposed.

3. Problem description

In this section we define and formulate the corresponding mathematical model as the foundation of the proposed robust PRP. The problem on hand is defined on a complete directed graph $G = (N, A)$ with $N = \{0, 1, 2, ..., n\}$ as the set of nodes that node 0 considered as a depot and N_0 as the set of costumers. $A = \{(i, j) : i, j \in N, i \leqslant j\}$ is the set of arcs and the distance from node i to node j is shown by d_{ij}. The number of homogeneous vehicles is a deterministic exogenous parameter and set of vehicles is represented by $k = \{1, 2, ..., m\}$, and the capacity of each vehicle is equal to Q. Each customer $i \in N_0$ has a non-negative demand q_i and a time interval $[a_i, b_i]$ in which service must start; early arrivals to customer nodes are permitted but a vehicle, arriving early must wait until time a_i before service can start. The service time of customer i is denoted by t_i.

The tilde ($\tilde{}$), bar ($\bar{}$) and hat ($\hat{}$) accents distinguish the uncertain parameters, nominal value (which is used in deterministic modeling), and uncertain part of each uncertain parameters. Therefore, $\tilde{q}_i$ is a uncertain parameter which is donated to a customer demand with nominal value of $\bar{q}_i$ and uncertain part of $\hat{q}_i$. An uncertainty interval of $\tilde{q}_i$ can be shown as $[\bar{q}_i - \hat{q}_i, \bar{q}_i + \hat{q}_i]$.

Uncertain demand results for a mixture of reasons; some variation in customer's interests, modes, styles, requirements, and the number of competitors faced in the market. These changes might affect the demand of the customer after transport order placement. As the same way, real-life issues (i.e., congestion, weather conditions) can lead to increase in traveling times. Longer traveling times due to traveling at lower speeds lead to increased fuel consumption and CO_2e emissions.

3.1. *Fuel consumption and CO_2e emissions*

The PRP model is introduced by Bektaş and Laporte (2011) based on the comprehensive emission model presented by Barth and Boriboonsomsin (2008), Barth et al. (2005), and Scora and Barth (2006). Recently, Demir et al. (2012) have extended the PRP with a more accurate fuel consumption rate function. In this formulation, the fuel consumption rate is calculated as the following:

$$F(v) = \lambda \left(kNV + w\gamma\alpha v + \gamma\alpha f v + \beta\gamma v^3 \right) d/v. \quad (1)$$

where v,τ,θ, d and w denote the vehicle speed, acceleration, road gradient, distance, and curb weight of an empty vehicle, respectively. To simplify notation, let $\lambda = \xi/(\kappa\psi)$ and $\gamma = 1/(1000 n_{tf}\eta)$ be constants, $\alpha = \tau + g\sin\theta + gC_r\cos\theta$ be a vehicle-arc specific constant and $\beta = 0.5C_d\rho A$ be a vehicle-specific constant. Also, notation f denotes the vehicle payload. The cost of fuel and CO_2e emissions can be estimated as $TC = F(v)f_c$, where f_c is the unit cost of fuel and CO_2e emissions. The definition of all parameters and their typical values are given in Table 3.

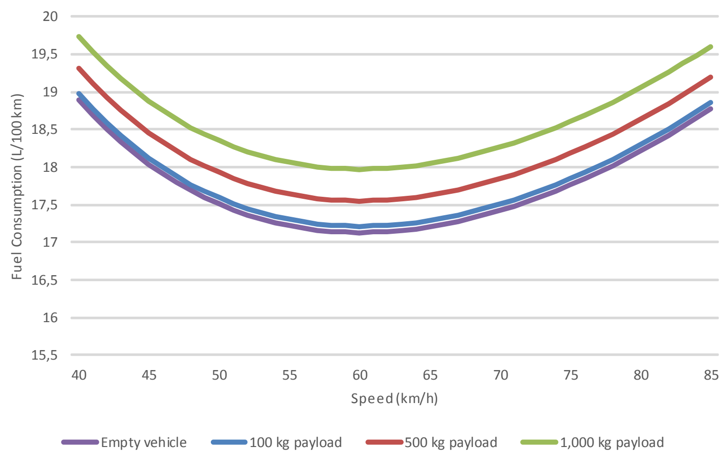
Table 3
Parameters used in the paper

Notation	Description	Typical values
w	curb-weight (kg)	6,350
ξ	fuel-to-air mass ratio	1
k	engine friction factor (kJ/rev/liter)	0.23
N	engine speed (rev/s)	37
V	engine displacement (liters)	5
g	gravitational constant (m/s^2)	9.81
C_d	coefficient of aerodynamic drag	0.7
ρ	air density (kg/m^3)	1.2041
A	frontal surface area (m^2)	3.912
C_r	coefficient of rolling resistance	0.01
n_{tf}	vehicle drive train efficiency	0.4
η	efficiency parameter for diesel engines	0.9
κ	heating value of a typical diesel fuel (kJ/g)	44
ψ	conversion factor (g/s to L/s)	737
v^l	lower speed limit (m/s)	11.1 (or 40 km/h)
v^u	upper speed limit (m/s)	23.6 (or 85 km/h)

We note that, particularly in freight transportation, the fuel consumption required to travel from one location to another is highly related to the carried load. The effect of payload on fuel consumption is visualized in Figure 1.

As seen from Figure 1, the fuel consumption mainly related to two important factors (i.e., travel speed and payload) under the same conditions (e.g., weather, traffic, driver). For low

Figure 1
Fuel consumption on 100-km road segment with different payload settings



speed values, the fuel consumption is very high because of the inefficiency in the usage of fuel. It decreases while speed goes up to a certain level, and then starts to increase because of the aerodynamic drag. Moreover, Figure 1 indicates that total vehicle weight has an impact on fuel consumption and increases with the increase in payload. Since these two factors are important, logistic service providers (LSPs) and freight forwarders should make better estimations for payload and travel speed in the planning of transport activities.

3.2. Mathematical formulation of the PRP

In this section we present a special case of the PRP to consider demand and travel time uncertainty. The model used here is the version proposed in Bektaş and Laporte (2011) and Demir et al. (2012). In order to investigate the effect of demand uncertainty along with travel time uncertainty, we adjust the PRP formulation by avoiding the driving wage component. This is done to investigate the actual fuel consumption with purely related to customer's demand and travel time. In the proposed formulation, we first discretize speed function defined by R non-reducing speed levels $\bar{v}^r$ $(r = 1, 2, , R)$. Binary variables x_{ij} are equal to 1 if arc (i, j) appears in a solution. z_{ij}^r is a binary variable equals 1 if arc $(i, j) \in A$ is crossed by a speed level r, and 0 otherwise. Continuous variables f_{ij} defines the total amount of flow on each arc. y_j is a non-negative continuous variable representing the time at that service starts at node $j \in N_0$.

An integer linear programming formulation of this variant of the PRP is shown below.

$$\text{Minimize} \quad \sum_{(i,j)\in\mathcal{A}} kNV\lambda d_{ij} \sum_{r=1}^{R} z_{ij}^{r}/\bar{v}^{r} \tag{2}$$

$$+ \sum_{(i,j)\in\mathcal{A}} w\gamma\lambda\alpha_{ij}d_{ij}x_{ij} \tag{3}$$

$$+ \sum_{(i,j)\in\mathcal{A}} \gamma\lambda\alpha_{ij}d_{ij}f_{ij} \tag{4}$$

$$+ \sum_{(i,j)\in\mathcal{A}} \beta\gamma\lambda d_{ij} \sum_{r=1}^{R} z_{ij}^{r}(\bar{v}^{r})^2 \tag{5}$$

Subject to

$$\sum_{j\in\mathcal{N}} x_{0j} = m \tag{6}$$

$$\sum_{j\in\mathcal{N}} x_{ij} = 1 \qquad \forall i \in \mathcal{N}_0 \tag{7}$$

$$\sum_{i\in\mathcal{N}} x_{ij} = 1 \qquad \forall j \in \mathcal{N}_0 \tag{8}$$

$$\sum_{j\in\mathcal{N}} f_{ji} - \sum_{j\in\mathcal{N}} f_{ij} = q_i \qquad \forall i \in \mathcal{N}_0 \tag{9}$$

$$q_j x_{ij} \leqslant f_{ij} \leqslant (Q - q_i)x_{ij} \qquad \forall (i,j) \in \mathcal{A} \tag{10}$$

$$y_i - y_j + t_i + \sum_{r\in\mathcal{R}} d_{ij} z_{ij}^{r}/\bar{v}^{r} \leqslant K_{ij}(1 - x_{ij}) \qquad \forall i \in \mathcal{N}, j \in \mathcal{N}_0, i \neq j \tag{11}$$

$$a_i \leqslant y_i \leqslant b_i \qquad \forall i \in \mathcal{N}_0 \tag{12}$$

$$\sum_{r=1}^{R} z_{ij}^{r} = x_{ij} \qquad \forall (i,j) \in \mathcal{A} \tag{13}$$

$$x_{ij} \in \{0,1\} \qquad \forall (i,j) \in \mathcal{A} \tag{14}$$

$$f_{ij} \geqslant 0 \qquad \forall (i,j) \in \mathcal{A} \tag{15}$$

$$y_i \geqslant 0 \qquad \forall i \in \mathcal{N}_0 \tag{16}$$

$$z_{ij}^{r} \in \{0,1\} \qquad \forall (i,j) \in \mathcal{A}, r = 1, ..., R. \tag{17}$$

The objective function (2)–(5) minimizes the quantity of total fuel consumption is derived from (1). Constraints (6) state that each vehicle must leave the depot. Constraints (7) and (8) are the degree constraints which ensure that each customer is visited exactly once. Constraints (9) and (10) define the arc flows. Constraints (11) and (12), where $K_{ij} = \max\{0, b_i + t_i + d_{ij}/v^l - a_j\}$, enforce the time window restrictions. Constraint (13) ensure that only one speed level is selected for each arc and $z_{ij}^r = 1$ if $x_{ij} = 1$. Finally, Constraints (14)–(17) enforce the binary and non-negativity restrictions on decision variables.

4. Modeling of the demand uncertainty

Stochastic optimization (SO) is one of the classical approaches to handle uncertainty, and it requires sufficient historical data to decide probability distribution of the parameters. This is not the case in many real world applications because it is hard to achieve accurate probability distribution function. According to robust optimization, a range of uncertain parameters is needed instead of finding the right probability distribution function. Interested readers are referred to Soyster (1973), Ben-Tal and Nemirovski (1998, 2000) and Bertsimas and Sim (2003, 2004) for more information on robust optimization.

As stated by Pishvaee et al. (2012), a solution to an optimization problem is said to be robust if it stays in feasible region for almost all possible values of uncertain parameters (i.e., feasibility robustness). It also results in objective function values which have minimum deviation from the planned optimal value for almost all possible values of uncertain parameters (i.e., optimality robustness). Therefore, a robust optimization (RO) approach tries to enhance the performance of the obtained solutions in both feasibility and optimality robustness aspects. As Pishvaee et al. (2012) mentioned, various RO approaches can be classified in three main categories, including (*i*) hard worst case (HWC), (*ii*) soft worst case (SWC), and (*iii*) realistic approaches. Among these three categories, the HWC approach provides the maximum conservatism against uncertainty for a decision maker. In other words, solution obtained with this approach is always feasible for all possible values of uncertain parameters. Regarding the value of objective function, this approach assures that the value of objective function never violates an optimal value. The HWC approach can be applied on different problems (e.g., VRP) in the area of logistics management. The SWC approach is a more flexible version of than the HWC approach and decision-makers can decide robustness level proportionally with their risk costs. Chance-constrained robust (CCR) model is another flexible version of the HWC approach with less variables and constraints.

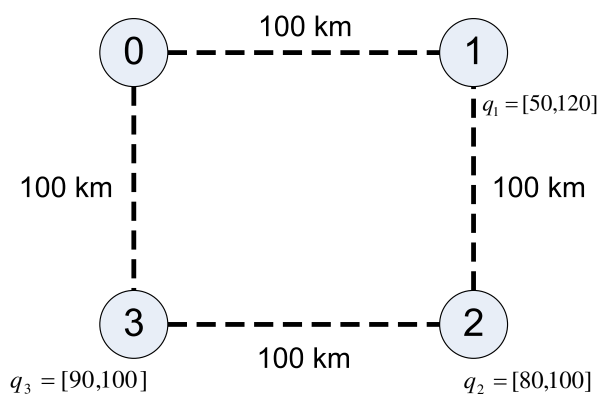
4.1. The hard worst case robust optimization approach

In this section, the HWC approach which is based on the formulation of Ben-Tal and Nemirovski (1998) is described. Sungur et al. (2008) developed the HWC robust optimization model based on the methodology introduced by Ben-Tal and Nemirovski (1998) to solve the CVRP with demand uncertainty and compared robust solutions with deterministic ones. The authors illustrated that an optimal solution of RVRP is the route that optimizes the worst case value over all uncertain data. The authors assumed that the maximum value of demand is realized for each customer and the found solution is feasible for each possible scenarios. Even though a solution is feasible for all scenarios, in the PRP and in other variants of cumulative VRPs, a found solution is not the worst objective function value. For the purpose of showing the difference, we now consider a four-node network as illustrated in Figure 2. The network forms a quadrangular shape in which the length of each sides is equal to one hundred distance unit. Assume that there is a single vehicle located at node 0 to serve three customers. The demand pattern of each customer is unknown and the upper and lower bounds of each customers' demand are set to $q_1 = [50, 120]$, $q_2 = [80, 100]$, $q_3 = [90, 100]$.

We now consider two traditional VRP objective functions. The first objective function is the minimization of the total distance, and the second objective function is the cumulative VRP minimizing the product of weighted load and distance.

$$\text{Model A: Minimize} \quad \sum_{(i,j)\in\mathcal{A}} d_{ij}x_{ij} \tag{18}$$

Figure 2
A sample four-node instance



Subject to
Constraints (6) to (10), (14) and (15).

$$\text{Model B: Minimize} \quad \sum_{(i,j)\in\mathcal{A}} d_{ij} f_{ij} \tag{19}$$

Subject to
Constraints (6) to (10), (14) and (15).

These models can be compared in two cases: (*i*) customers need to be serviced considering their upper bound, and (*ii*) customers need to be serviced considering their lower bound. In both models, unmet demands are not allowed (i.e., the maximum demand value for each customer). In both cases, model A yields two optimal routes, i.e., $(0, 1, 2, 3, 0)$ and $(0, 3, 2, 1, 0)$ of length 400 units, but model B has different optimal route in each case. Detailed results are shown in Table 4.

Table 4
An analysis of two routes with model B

Route 1					Route 2				
Arc	Load (i)	Load (ii)	Objective value (i)	Objective value (ii)	Arc	Load (i)	Load (ii)	Objective value (i)	Objective value (ii)
(0, 1)	320	320	32,000	32,000	(0, 3)	320	320	32,000	32,000
(1, 2)	200	270	20,000	27,000	(3, 2)	220	230	22,000	23,000
(2, 3)	100	190	10,000	19,000	(2, 1)	120	150	10,000	15,000
(3, 0)	0	100	0	10,000	(1, 0)	0	100	0	10,000
Total			62,000	82,000				64,000	80,000

Table 4 provides optimal results for two different solutions. Results show that by considering the effect of weight, the optimum solution may be different in each scenario. The first route (i.e., $0, 1, 2, 3, 0$) is is optimum for case (i) with 62,000 units, and the second route (i.e., $0, 3, 2, 1, 0$) is optimum solution for case(ii) with 80,000 units. According to this observation, the robust model introduced by Sungur et al. (2008) is not respondent for the HWC modeling of the cumulative VRP. In order to handle cumulative variables as in the PRP, we now reformulate the HWC robust approach as follows.

Minimize (2)-(3), (5) and:

$$\sum_{(i,j)\in\mathcal{A}} \gamma\lambda\alpha_{ij}d_{ij}f'_{ij} \tag{20}$$

Subject to

Constraints (6)–(8),(11)–(17) and:

$$\sum_{j\in\mathcal{N}} f'_{ji} - \sum_{j\in\mathcal{N}} f'_{ij} = \bar{q}_i - \hat{q}_i \qquad \forall i \in \mathcal{N}_0 \tag{21}$$

$$\sum_{j\in\mathcal{N}} f_{ji} - \sum_{j\in\mathcal{N}} f_{ij} = \bar{q}_i + \hat{q}_i \qquad \forall i \in \mathcal{N}_0 \tag{22}$$

$$f_{ij}x_{ij} \leqslant (Q - \bar{q}_i - \hat{q}_i)x_{ij} \qquad \forall (i,j) \in \mathcal{A} \tag{23}$$

$$f_{ij}x_{ij} \geqslant (\bar{q}_j + \hat{q}_j)x_{ij} \qquad \forall (i,j) \in \mathcal{A} \tag{24}$$

$$f_{0i}x_{ij} \leqslant f'_{0i} \qquad \forall i \in \mathcal{N}_0 \tag{25}$$

$$\sum_{(i,j)\in\mathcal{A}} f_{ji} - \sum_{(i,j)\in\mathcal{A}} f_{ij} - \sum_{(i,j)\in\mathcal{A}} f'_{ji} + \sum_{(i,j)\in\mathcal{A}} f'_{ij} = 2\hat{q}_i \qquad \forall i \in \mathcal{N} \tag{26}$$

$$f'_{ij} \geqslant 0 \qquad \forall (i,j) \in \mathcal{A}, \tag{27}$$

where continuous variables f'_{ij} are defined as the total amount of flow on arc (i, j) if each customer receives a lower bound of demand when vehicles are loaded with an upper bound of demand. Variables f_{ij} show the total amount of flow on arc (i, j) if each customer receives an upper bound of demand when vehicles are loaded with an upper bound of demand. The objective function (20) is counterpart of the objective function (4) in robust approach. Constraints (21) and (22) define the hard worst case arc flows as well as eliminating sub-tours like constraints (9). Constraints (23) and (24) enforce the capacity limitations on payload. Constraints (25)-(26) balance the flow and ensure the relations between variables f_{ij} and f'_{ij}.

4.2. The soft worst-case robust optimization approach

In order to control the conservation of solution, a new parameter ψ can be used as an *uncertainty budget*. The concept of *uncertainty budget* or *the price of robustness* was introduced by Bertsimas and Sim (2003, 2004). In the proposed SWC robust optimization approach, parameter ψ can be selected as any value in the interval of $[0, |N_0|]$, which $\psi = |N_0|$ is the most conservative condition. The SWC approach is a more flexible version of the HWC approach, and tries to minimize the worst case value of objective function without satisfying all constraints in their extreme worst case values (see, e.g., Pishvaee et al. (2012)). Moreover, binary variables Γ_i are defined to control the degree of conservation in each constraint i in which Γ_i is equal to 1 if $\tilde{q}_i$ receives its upper bound value (otherwise it takes 0). Therefore, $\sum_i \Gamma_i = \psi$ means that only ψ number of uncertain parameters ($\tilde{q}_i$) achieve the corresponding upper bound values. According to above-mentioned descriptions the SWC, a new robust approach can be formulated as follows.

Minimize (2)–(3), (5) and (20):
Subject to
Constraints (6)–(8), (11)–(17), (21),(25), (27) and:

$$\sum_{j\in\mathcal{N}} f_{ji} - \sum_{j\in\mathcal{N}} f_{ij} = \bar{q}_i + \Gamma_i \hat{q}_i \qquad \forall i \in \mathcal{N}_0 \tag{28}$$

$$f_{ij}x_{ij} \leqslant (Q - \bar{q}_i - \Gamma_i \hat{q}_i)x_{ij} \qquad \forall (i,j) \in \mathcal{A} \tag{29}$$

$$f_{ij}x_{ij} \geqslant (\bar{q}_j + \Gamma_i \hat{q}_j)x_{ij} \qquad \forall (i,j) \in \mathcal{A} \tag{30}$$

$$\sum_{(i,j)\in\mathcal{A}} f_{ji} - \sum_{(i,j)\in\mathcal{A}} f_{ij} - \sum_{(i,j)\in\mathcal{A}} f'_{ji} + \sum_{(i,j)\in\mathcal{A}} f'_{ij} = (1 + \Gamma_i)\hat{q}_i \qquad \forall i \in \mathcal{N} \tag{31}$$

$$\sum_{i\in\mathcal{N}_0} \Gamma_i = \psi \tag{32}$$

$$\Gamma_i \in \{0,1\} \qquad \forall i \in \mathcal{N}_0, \tag{33}$$

where binary variables Γ_i are equal to 1 if an upper bound of customer i corresponding to the demand allocated. And it is equal to 0 if nominal demand allocated. Constraints (28) to (31) are counterpart of constraints (22) to (24) and (26). Constraints (32) ensure that only ψ number of customers receive an upper bound value of the corresponding uncertain parameters. In this problem the maximum value of ψ is equal to the total number of nodes. Time windows constraints are used as in hard worst case approach. Due to the multiplication of binary variables in constraints (29) and (30), the SWC formulation becomes a non-linear programming problem. To convert the formulation into its equivalent linear form, we define two new set of binary variables as follows.

$$\nu_{ij} = \Gamma_i x_{ij} \qquad \forall (i,j), \quad \text{and}$$

$$\nu'_{ij} = \Gamma_j x_{ij} \qquad \forall (i,j).$$

Hence, constraints (29) and (30) can be replaced by following constraints:

$$f_{ij}x_{ij} \leqslant (Q - \bar{q}_i)x_{ij} - \hat{q}_i \nu_{ij} \qquad \forall (i,j) \in \mathcal{A} \tag{34}$$

$$f_{ij}x_{ij} \geqslant \bar{q}_j x_{ij} + \hat{q}_j \nu'_{ij} \qquad \forall (i,j) \in \mathcal{A} \tag{35}$$

$$\nu_{ij} \leqslant x_{ij} \qquad \forall (i,j) \in \mathcal{A} \tag{36}$$

$$\nu_{ij} \leqslant \Gamma_i \qquad \forall (i,j) \in \mathcal{A} \tag{37}$$

$$\Gamma_i + x_{ij} - \nu_{ij} \leqslant 1 \qquad \forall (i,j) \in \mathcal{A} \tag{38}$$

$$\nu'_{ij} \leqslant x_{ij} \qquad \forall (i,j) \in \mathcal{A} \tag{39}$$

$$\nu'_{ij} \leqslant \Gamma_j \qquad \forall (i,j) \in \mathcal{A} \tag{40}$$

$$\Gamma_j + x_{ij} - \nu'_{ij} \leqslant 1 \qquad \forall (i,j) \in \mathcal{A} \tag{41}$$

$$\nu_{ij} \in \{0,1\} \qquad \forall (i,j) \in \mathcal{A} \tag{42}$$

$$\nu'_{ij} \in \{0,1\} \qquad \forall (i,j) \in \mathcal{A}. \tag{43}$$

4.3. The chance-constrained robust optimization approach

If a probability distribution of demands is known, we can adapt a new robust formulation based on chance constrained programming. When the objective is to have a solution that is feasible at least for (1-α) percent of scenarios, the HWC formulation can be converted to chance-constrained robust (CCR) formulation with change constraints (21) to (24) and (26). If an uniform distribution is considered for demand-related parameters, the CCR formulation can be given as follows.

Minimize (2)–(3), (5) and (20):

Subject to

Constraints (6)–(8), (11)–(17), (21), (25), (27) and:

$$\sum_{j\in\mathcal{N}} f'_{ji} - \sum_{j\in\mathcal{N}} f'_{ij} = \bar{q}_i - (1-\alpha)\hat{q}_i \qquad \forall i \in \mathcal{N}_0 \tag{44}$$

$$\sum_{j\in\mathcal{N}} f_{ji} - \sum_{j\in\mathcal{N}} f_{ij} = \bar{q}_i + (1-\alpha)\hat{q}_i \qquad \forall i \in \mathcal{N}_0 \tag{45}$$

$$f_{ij}x_{ij} \leqslant (Q - \bar{q}_i - (1-\alpha)\hat{q}_i)x_{ij} \qquad \forall (i,j) \in \mathcal{A} \tag{46}$$

$$f_{ij}x_{ij} \geqslant (\bar{q}_j + (1-\alpha)\hat{q}_j)x_{ij} \qquad \forall (i,j) \in \mathcal{A} \tag{47}$$

$$\sum_{(i,j)\in\mathcal{A}} f_{ji} - \sum_{(i,j)\in\mathcal{A}} f_{ij} - \sum_{(i,j)\in\mathcal{A}} f'_{ji} + \sum_{(i,j)\in\mathcal{A}} f'_{ij} = 2(1-\alpha)\hat{q}_i \qquad \forall i \in \mathcal{N}. \tag{48}$$

5. Computational experiments

This section presents the results of extensive computational experiments performed to assess the performance of our three robust approaches. We first describe the PRPLIB instances and then present the results.

5.1. Data and experimental setting

We have used specifically designed PRPLIB instances which are introduced by Demir et al. (2012). These PRPLIB instances are generated to analyze the fuel consumption in more realistic transport networks. The instances represent randomly selected cities from the United Kingdom and therefore use real geographical distances. Time windows and service times are randomly generated. In this paper, we only consider three sets of five instances each. The size of the considered PRPLIB instances in this paper ranges from 10 to 20 customers' nodes. All instances are available for download from the website http://www.apollo.management.soton.ac.uk/prplib.

The proposed robust solution formulations were implemented in an Advanced Integrated Multidimensional Modeling Software (AIMMS) (Roelofs, 2010). All experiments were conducted on a computer with core i5-4200U 1.6GHz CPU and 8 GB RAM. A common time-limit of three hours was imposed on the solution time for all instances.

In Table 5, for the SWC robust approach, different robustness levels and the corresponding values of (ψ) are shown. The first column indicates an instance set based on number of nodes in each instance and the columns 2 to 5 show the value of (ψ) for different levels of robustness. It is noted that the column "Zero" represents the zero robustness level, "Low" represents the low robustness level, "Medium" represents the medium robustness level, and "High" represents the high robustness level.

To help assess the complexity of the robust approaches, we now present the average time required by the AIMMS to solve an instance to optimality in Table 6. For each instance set, we present the instance set name in column "Instance set", the uncertainty level of demand in column "Uncert. level", and average solution times required for each approach in the following seven columns. Moreover,"HWC" represents hard worse case solution and"LSL" represents

Table 5
Different robustness level and corresponding values of (ψ) in each instance type

Instance set	Value of (ψ) for different robustness levels			
	Zero	Low	Medium	High
UK10	0	2	5	7
UK15	0	3	7	11
UK20	0	5	10	15

the low safety level (α=0.1), and "'HSL" represents the high safety level (α=0.01) in chance-constrained approach. Uncertainty level for (10–50%) is an average value of these bounds (i.e. UK10 (10–50 %)=(UK10 (10%) + UK10 (50%))/2).

Table 6
Averege CPU time required to solve three sets of PRPLIB instances

Instance set	Uncert. level	SWC				HWC	CCR	
		Zero	Low	Medium	High		LSL	HSL
UK10	10%	3.0	3.0	4.2	4.2	1.9	2.6	3.1
UK15	10%	53.1	104.5	93.4	144.5	68.4	54.0	115.2
UK20	10%	394.3	3,944.5	2,820.3	3,759.4	1,372.4	1,803.2	879.1
UK10	50%	2.7	4.9	4.9	14.0	6.1	10.8	13.5
UK15	50%	119.4	149.4	228.4	250.2	312.9	249.8	351.4
UK20	50%	3,002.7	5,335.0	6,752.5	10,406.3	7,495.7	8,400.4	10,800.0
UK10	10–50%	2.8	3.9	4.5	9.1	4.0	6.7	8.3
UK15	10–50%	86.2	127.0	160.9	197.4	190.6	151.9	233.3
UK20	10–50%	1,698.5	4,639.7	4,786.4	7,082.8	4,434.1	5,101.8	5,839.5
Average		595.9	1,590.2	1,650.6	2,429.8	1,542.9	1,753.5	2,027.0

Table 6 provides the average CPU time required by the solver for three instance category (i.e., 10-, 15-, and 20-node instances). The average times required with the SWC ranges from 595.9 to 2,429.8 seconds. The average time required with the HWC is around 1,542.9 seconds. The average times required with the LSL and HSL are found to be 1,753.5 and 2,027 seconds, respectively.

5.2. Results of the low uncertainty level

Fuel consumption depends on several factors, such as vehicle speed, vehicle load, etc (Demir et al., 2011). In this section, we particularly look at the low demand uncertainty and its impact on fuel consumption. More specifically, we set the low uncertainty level as 10% for each robust strategy.

With the HWC approach and low uncertainty level, we now provide a numerical comparison between deterministic and robust solutions on instance UK15_01 in Table 7. In this example, we provide the routes, the total distance traveled, the number of vehicles used, and the total fuel consumption consumed.

Table 7 provides that robust solution leads to most reliable results with limited increase in fuel consumption. We suppose that each customer's demand follows the normal distribution, and therefore each route should satisfy total demand with the normal distribution. With the

Table 7
Two solutions for UK15_01

Solutions	Routes	Total distance (m)	# of vehicles	Fuel consumption (L)
Deterministic solution	0–8–6–10–7–1–9–3–0 0–12–14–13–4–5–2–15–11–0	709,035	2	132.21
Robust solution	0–8–6–14–13–4–5–0 0–9–3–15–2–11–0 0–12–10–7–1–0	727,303	3	135.03

deterministic model and nominal value for customer's demand, optimum solution has two routes. In each route, vehicle maybe confronted with shortage by probability equal to 0.5 and finally, deterministic solution feasible with probability equal to 0.25. On the other hand, robust solution is feasible, in all situations, with only 2.58% increase in total distance (709,035 to 727,303 meters), 2.13% in fuel consumption (132.21 to 135.03 liters), and 8.9 kg more CO_2e emissions.

We now present the detailed results obtained for all robust scenarios with three different approaches. For each instance, we present the optimal fuel consumption (in L) in column "Optimal", and the difference between optimal and robust solutions values (in L) in the following seven columns in Table 8.

Table 8
Fuel consumption comparison among different robust strategies for the low uncertainty level

Instance	Optimal	SWC				HWC	CCR	
	(L)	Zero	Low	Medium	High		LSL	HSL
UK10_01	104.55	0.80	0.95	1.30	2.62	3.21	2.98	3.18
UK10_02	133.32	0.90	1.06	1.36	1.67	2.41	2.16	2.38
UK10_03	125.69	0.56	0.65	0.89	1.12	1.64	1.48	1.63
UK10_04	120.26	0.79	0.89	1.17	1.48	2.09	1.88	2.06
UK10_05	111.32	0.56	0.65	0.89	1.16	1.65	1.49	1.64
UK15_01	185.09	1.59	1.79	2.30	2.94	3.94	3.62	3.92
UK15_02	129.98	1.07	1.19	1.51	2.00	2.76	2.47	2.72
UK15_03	179.88	1.15	1.28	1.67	2.30	3.12	2.80	3.08
UK15_04	189.12	1.39	1.67	2.44	5.00	6.96	5.78	6.92
UK15_05	188.41	1.26	1.47	1.99	2.86	17.24	16.82	17.20
UK20_01	199.05	1.66	1.96	2.57	3.39	14.24	4.03	14.18
UK20_02	211.88	1.59	1.80	2.28	3.23	4.66	4.20	4.61
UK20_03	117.30	1.02	1.18	4.29	10.20	11.02	10.73	10.99
UK20_04	204.04	1.86	2.30	3.02	4.09	10.60	10.21	10.56
UK20_05	183.74	1.45	1.72	2.14	2.91	4.33	3.91	4.27
Average		1.18	1.37	1.99	3.13	5.99	4.97	5.96

Table 8 shows that the difference in total fuel consumption obtained with robust approaches are positive, meaning that robust solutions require only limited increase in terms of fuel consumption to complete the transport plans. The average increase in fuel consumption with the SWC are 1.18, 1.37, 1.99 and 3.13 liters for the corresponding robust strategies. The average increase in fuel consumption with the HWC is 5.99 liters. The average increase in fuel consumption with the LSL and HSL are found to be 4.97 and 5.96 liters, respectively.

5.3. Results of the high uncertainty level

In this section, we assume a high uncertainty of demand and present the detailed results for each instance. More specifically, we set the high uncertainty level as 50% for each robust strategy. The detailed results on fuel consumption (in L) are provided in Table 9.

Table 9
Fuel consumption comparison among different robust strategies for the high uncertainty level

Instance	Optimal	SWC				HWC	CCR	
	(L)	Zero	Low	Medium	High		LSL	HSL
UK10_01	104.55	4.03	4.82	8.15	16.11	29.28	28.38	29.18
UK10_02	133.32	4.51	5.31	6.84	15.75	18.10	17.29	18.01
UK10_03	125.69	2.83	3.29	4.47	5.67	8.28	7.44	8.18
UK10_04	120.26	3.98	4.50	5.89	7.49	15.03	14.25	14.94
UK10_05	111.32	2.87	3.33	4.53	6.10	11.17	10.24	11.07
UK15_01	185.09	5.92	6.43	8.52	12.40	36.51	34.88	36.27
UK15_02	129.98	5.34	5.93	7.57	10.01	20.86	19.70	20.73
UK15_03	179.88	5.78	6.45	8.41	11.64	39.38	20.12	39.22
UK15_04	189.12	6.67	7.58	11.68	22.93	49.02	46.12	47.55
UK15_05	188.41	6.38	7.46	11.35	29.75	36.68	35.07	36.51
UK20_01	199.05	8.35	9.92	20.30	34.02	59.51	49.19	59.29
UK20_02	211.88	7.96	9.08	12.33	37.96	57.60	55.62	57.37
UK20_03	117.30	5.16	6.06	10.59	21.79	33.60	31.44	33.47
UK20_04	204.04	9.38	11.34	17.66	26.08	52.98	43.89	52.75
UK20_05	183.74	7.15	8.65	13.61	25.81	50.27	39.71	50.09
Average		5.75	6.68	10.13	18.90	34.55	30.22	34.31

Table 9 shows that the difference in total fuel consumption obtained with robust approaches increases with the increase in uncertainty level. The average increase in fuel consumption with the SWC are 5.78, 6.68, 10.13 and 18.90 liters for the corresponding robust strategies. The average increase in fuel consumption with the HWC is 34.55 liters. The average increase in fuel consumption with the LSL and HSL are 30.22 and 34.31 liters, respectively. According to these results, we can claim that the demand uncertainty plays a vital role in transport planning and it must be considered during the planning phase. As shown, robust solutions lead to most reliable solutions with very limited increase in the objective function related to fuel consumption and CO_2-equivalent emissions. If demand uncertainty is not considered, LSPs and freight forwarders do not only lose money but also produce more CO_2e emissions for their transport activities.

5.4. The effect of the travel time uncertainty

Different countries impose different restrictions on traveling speed. Moreover, traveling speed might be restricted based on other factors, such as congestion, weather conditions, etc (Demir et al., 2015). Congestion happens in transportation networks when entities compete individually for a limited capacity. Congestion causes increased travel times, operating costs, and unreliability in travel activities (Banfi et al., 2000). Congestion could also indirectly result in increased fuel. Since we discretize traveling speed, we can adjust these limits and observe the uncertainty of travel times with the hard-worst case robust approach. As explained in section 3.2, speed levels R (i.e., $R = 40, ..., 85$ km/h) can be redefined for both low and high uncertainty levels. The low travel uncertainty is defined as $R = 40, ..., 75$ km/h whereas the high travel uncertainty (i.e., high congestion) is defined as $R = 40, ..., 55$ km/h. The detailed results are provided in Table 10.

Table 10
Results of travel time uncertainty on fuel consumption

Instance	Optimal (L)	High travel time uncertainty		Low demand and high travel time uncertainty		High demand and high travel time uncertainty	
		Difference	# of vehicles	Difference	# of vehicles	Difference	# of vehicles
UK10_01	104.55	1.40	2	3.73	2	29.90	3
UK10_02	133.32	2.28	2	4.76	2	18.81	3
UK10_03	125.69	0.62	3	2.26	3	8.90	3
UK10_04	120.26	0.59	2	2.68	2	15.66	3
UK10_05	111.32	0.55	2	2.19	2	11.73	2
UK15_01	185.09	1.71	3	5.02	3	37.53	4
UK15_02	129.98	0.64	2	3.40	2	21.56	3
UK15_03	179.88	0.88	3	4.01	3	40.39	4
UK15_04	189.12	1.70	4	7.90	4	50.13	5
UK15_05	188.41	9.82	3	22.29	3	37.71	3
UK20_01	199.05	0.97	3	19.77	4	60.68	4
UK20_02	211.88	1.03	3	5.69	3	58.83	4
UK20_03	117.30	0.57	3	11.63	3	34.29	4
UK20_04	204.04	0.99	3	11.63	4	54.15	5
UK20_05	183.74	0.92	3	6.46	3	51.33	5
Average		1.64	2.73	7.56	2.87	35.44	3.67

Table 10 shows the results in three category. The first category is presented for high travel time uncertainty. In this case, demand uncertainty is not considered. The second category presents results for high travel time with low demand uncertainty. Finally, the last category presents results for high travel time with high demand uncertainty. Moreover, we provide the number of vehicles used in the solutions in Table 10. It is noted that low travel time uncertainty provided the similar results with optimal values because of the loose time windows of the PRPLIB instances. However, one can still obtain realizable solutions for the least fuel consumption with both high travel time and demand uncertainty.

6. Conclusions and future research directions

Vehicle routing problems have been conventionally optimized with a special focus on distance minimization or the number of vehicles reduction. With an ever growing concern for the environment, better planning algorithms were required by the logistic service providers or freight forwarders. As a result, in recent years, there have been emerging solutions provided by scientific researchers as well as practitioners to minimize fuel consumption and particularly CO_2e emissions. In this paper, we have selected the well-known green vehicle routing problem formulation, namely Pollution-Routing Problem (PRP), and extended it to consider data uncertainties. In particular, we have formulated three different robust optimization approaches for the solution of the PRP with demand and travel time uncertainty.

Almost all studies address the green vehicle routing problems in a context where all parameters are known with certainty. By using several robust techniques, we have showed that it is possible to deal with data uncertainty and to obtain reliable solutions. However, this might cause additional up to 30 L of fuel for a 10-node instance, up to 50 L of fuel for a 15-node instance, and up to 60 L of fuel for a 20-node instance. With the help of robust techniques, we can decrease the risk of shortage and unmet demand, which might require more costly solutions than robust ones in practice (i.e., extra fuel and time due to using an additional vehicle). Decreasing the uncertainty level will lead to closer results to the corresponding deterministic ones. Similarly, increase in the robustness level will lead to higher costs. Logistics companies need to address their operations with better planning algorithms and pricing policies to enable an efficient and sustainable freight transportation system. By looking at the results of robust optimization approaches, the following two areas are identified as further research directions. We have shown that a majority of the existing studies focus only on the routing aspect of green vehicle routing. Other problems which can be linked to routing may offer former reductions in emissions, such as location-routing problems, where relocating the depot or locating alternative facilities may reduce the overall emissions. Another possible future research direction could be related to the use of alternative types of vehicles, and in particular electric vehicles, which make use of new technologies

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